Construction Financial Management

S. L. Tang

S.L. Tang

Construction Financial Management

Contents

Download free eBooks at bookboon.com

1 Financial Statements – Income Statement and Balance Sheet

1.1 Introduction

In all businesses, financial statements are important for reflecting the financial health of a company. The construction industry is without exception. Among various financial statements, two of them are the most important ones. They are (1) Income Statement (or called Profit & Loss Account) and (2) Balance Sheet. In this chapter, these two important financial statements will be discussed in detail. First, we will look at an income statement. Second, we will see the use of a balance sheet. Third, we will see other important items that are closely related to these two financial statements.

1.2 Income Statement

An 'income statement' shows the profit made or the loss incurred by a company in a period of time (usually one year), and that is why it is sometimes also called a 'profit and loss account'. In an income statement, usually two columns of figures are shown for comparison purpose, one column showing the figures of the most recent year and the other column showing that of the year before. An example of income statement is shown in Table 1.1 (see next page). It shows a company's revenue, costs and other expenses including the interest payment on loan (if there is a loan borrowed by the company) and the income tax paid. The purpose of the income statement is to show whether or not a company's business is profitable.

The first row of the income statement shown in Table 1.1 is Revenue (some people may use the terms such as Sales or Income). It represents the amount of money the company receives before (or without) deducting any expenditures related to the company's revenue. The second row is Cost of Revenue. It is the direct construction/production cost etc. which the construction company has incurred in order to earn the Revenue. We obtain Gross Profit by deducting the Cost of Revenue from the Revenue. Operating Expenses usually consist of Variable Overhead (e.g. advertising, plant, equipment, vehicles, etc.) and Fixed Overhead (e.g. depreciation, rent, salaries, insurance, etc.), a more detailed example of which will be given in Table 2.1 of Chapter 2. After deducting the Operating Expenses from the Gross Profit, we obtain Operating Profit.

The Interest Expense shown under Other Income/Expense in Table 1.1 is the interest payment on loan if a loan is borrowed by the company. Since interest on loan is tax deductible so it must be deducted in order to obtain the Net Profit before Tax, which is calculated by deducting the Total Other Income/ Expense from the Operating Profit. The amount of income tax payable is then calculated based on the tax rate. By deducting this calculated Tax Expense from the Net Profit before Tax, we obtain Net Profit after Tax. This figure indicates whether or not the construction company's business is profitable.

Income Statement		
	Period (one year) ended on	
	31/12/2012	31/12/2011
Revenue	40,185,000	38,483,000
Cost of Revenue		
Materials	13,000,000	12,500,000
Labour	5,500,000	5,400,000
Subcontracts	12,500,000	12,000,000
Other direct costs	1,087,000	1,085,000
Total Cost of Revenue	32,087,000	30,985,000
Gross Profit	8,098,000	7,498,000
Operating Expenses		
Variable overhead	2,036,500	1,943,500
Fixed overhead	3,358,500	2,979,500
Total Operating Expenses	5,395,000	4,923,000
Operating Profit	2,703,000	2,575,000
Other Income/Expense		
Gain/loss on sale of assets	30,000	(38,000)
Miscellaneous income/expense	(5,500)	4,000
Interest income	19,000	12,900
Interest expense	(42,500)	(41,000)
Total Other Income/Expense	1,000	(62,100)
Net Profit before Tax	2,704,000	2,512,900
Tax Expense (25% tax rate)	676,000	628,225
Net Profit after Tax	2,028,000	1,884,675

Table 1.1 – Example of Income Statement

1.3   Balance Sheet

A balance sheet shows a company's financial position as at a point of time (usually the last date of the company's fiscal year), like a snap shot picture of the company's financial situation at that particular time point. An example of balance sheet of a construction company is shown in Table 1.2 (see next page). There are three major items in a balance sheet: (1) Assets, (2) Liabilities, and (3) Equity (or called Net Worth). These will be explained in the following sub-sections.

1.3.1　　Assets

Assets are classified into two categories: (1) Current Assets and (2) Fixed Assets. They represent what a company owns, and are usually presented at the top (first part) of a balance sheet. Current Assets of a construction company are usually cash, accounts receivable, construction material inventory and so on which have high liquidity (i.e. can be turned into cash easily). Fixed Assets of a construction company are usually property and equipment, construction plant, trucks and so on which cannot be readily turned into cash in a short time. Fixed Assets are also called Long-term Assets. When Current Assets and Fixed Assets are added together, the sum is called Total Assets.

1.3.2　　Liabilities

When a construction company owes obligations to some third parties, we call these obligations Liabilities. Liabilities are usually presented at the middle part of a balance sheet. They are classified into (1) Current Liabilities and (2) Long-term Liabilities. Current Liabilities of a construction company include bank overdraft and short term bank loan, accounts payable to subcontractors, suppliers and employers, rents, utilities and so on. They are debts the company has to pay, say, within a year. Long-term Liabilities, however, are obligations with a period more than one year, usually a few years or even longer. They often refer to long term bank loans or loans for mortgages of equipment, building, land, or even cars/trucks. These long-term debts are usually repaid by installments. When Current Liabilities and Long-term Liabilities are added together, the sum is called Total Liabilities.

1.3.3　　Equity

Equity is the capital invested by the owner(s) of a company. Because some construction companies are owned by stockholders, so Equity is sometimes also referred to as Stockholders' Equity. Another name for it is 'Net Worth'. Equity represents the Net Worth of the business, and can be calculated by summing up the capital the owners have invested and the profits that have been accumulated (after deducting all the dividends paid so far to the owners) and retained up to the present moment since the business began (see Table 7.12 of Chapter 7).

Balance Sheet		
	As at	
	31/12/2012	31/12/2011
Assets		
Current Assets		
Cash	2,589,000	1,967,890
Accounts receivable	5,767,000	5,403,670
Retention money	1,641,750	1,350,918
Material Inventory	850,000	520,000
Costs and estimated earnings in excess of billings on work in progress	547,250	450,306
Prepaid expenses and others	894,500	983,944
Total Current Assets	12,289,500	10,676,728
Fixed assets		
Property and equipment	15,536,900	13,800,000
Construction plant	2,680,040	2,039,480
Vehicles/Trucks	2,070,000	1,812,000
Furniture and fixtures	345,000	379,000
Total depreciable assets	20,631,940	18,030,480
Less accumulated depreciation	12,529,373	11,158,000
Net Fixed Assets	8,102,567	6,872,480
Total Assets	20,392,067	17,549,208
Liabilities		
Current Liabilities		
Accounts payable	4,325,250	4,773,240
Accrued expenses	1,586,037	1,475,918
Notes payable	647,250	491,973
Retention money payable	919,380	756,514
Billings in excess of costs and estimated earnings on work in progress	617,205	678,922
Other current liabilities	355,713	292,699
Total Current Liabilities	8,450,835	8,469,266
Long-term Liabilities	3,528,557	3,695,267
Total Liabilities	11,979,392	12,164,533
Equity (i.e. Net Worth)		
Capital stock	3,500,000	2,500,000
Additional paid-in capital	1,000,000	1,000,000
Retained earnings	3,912,675	1,884,675
Total Equity	8,412,675	5,384,675
Equity + Total Liabilities	20,392,067	17,549,208

Table 1.2 – Example of Balance Sheet

The following is a very important mathematical relationship:

Equity + Total Liabilities = Total Assets   (Eq 1.1)

(Net Worth)

Equation 1.1 is also called "The Accounting Equation". Readers can find that the last row of Table 1.2 is equal to the Total Assets' row at the middle of the balance sheet. The name, Balance Sheet, is in fact derived from this fact – they must be in balance.

1.4   Working Capital and Current Ratio

1.4.1   Working Capital

The difference of Current Assets and Current Liabilities is called Working Capital. So, it can be mathematically presented as:

Working Capital = Current Assets – Current Liabilities   (Eq 1.2)

In our example (Table 1.2), the working capital is $2,207,462 (i.e. $10,676,728 – $8,469,266) as at 31 Dec 2011, and is $3,838,665 (i.e. $12,289,500 – $8,450,835) as at 31 Dec 2012.

The amount of working capital in hand is a measure of the short term financial strength of a construction company. It is because the liquidity (see section 1.3.1) of working capital is high. Current assets are used to pay current liabilities, and therefore it is important to know how much current assets exceed current liabilities, which is told by the amount of working capital. Working capital increases when a company makes a profit on a project, sells equipment or other assets, or has a long term loan from a bank. A long term bank loan can increase current (short term) assets, but at the same time increases long term liabilities. However, construction companies usually do not easily resort to selling equipment or borrowing long term loans. Working capital decreases when a company loses money on a project, or when it purchases equipment, or makes repayments on long term loans. Construction companies are, however, eager to purchase equipment or to pay off loans.

Empirically, it has been found that the volume of unfinished work of all projects in hand should be at most about ten times the working capital for a construction company. If there is a big project among the projects in hand, the unfinished volume of work of this big project should not be more than five times the working capital. These are to ensure that the company may maintain a healthy financial condition.

1.4.2   Current Ratio

Besides the working capital, we also use a ratio for identifying a construction company's liquidity (or its ability to fulfill short term financial obligations). The ratio is called Current Ratio and is defined as follows:

Current Ratio = Current Assets / Current Liabilities   (Eq 1.3)

In our example (Table 1.2), the current ratio is 1.26 (i.e. $10,676,728 / $8,469,266) as at 31 Dec 2011, and is 1.45 (i.e. $12,289,500 / $8,450,835) as at 31 Dec 2012.

The current ratio for a construction company should be 1.3 or higher. This is obtained by experience over a long period of time. In our example, the financial health of the company was a little unsatisfactory in 2011, because the current ratio was 1.26, a little lower than 1.3. However, the financial situation became more healthy in 2012, because the current ratio increased to 1.45, higher than 1.3.

In financial management, there are many ratios other than the current ratio. We will look at them, particularly those related to construction, in Chapter 2.

1.5   Under Billing and Over Billing

Under billing is expressed in the balance sheet as "Costs and estimated earnings in excess of billings on work in progress" under Current Assets (see Table 1.2). Over billing is expressed as "Billings in excess of costs and estimated earnings on work in progress" under Current Liabilities. Let us look at an example of what they are and how they are evaluated.

A construction company has the following project financial data:

- Contract sum $8,000,000
- Billed to date $4,700,000
- Cost incurred (i.e. cost of revenue) to date $3,700,000
- Estimated cost to complete $3,000,000

Up to the present moment, the percentage of completion

= [Cost incurred to date / (Cost incurred to date + Estimated cost to complete)] × 100%

= [$3,700,000 / ($3,700,000 + $3,000,000)] × 100%

= 55.22%

Revenue to date = (Contract sum) × (% of completion)

= $8,000,000 × 55.22% = $4,417,600

Gross Profit to date = Revenue to date – Cost of Revenue to date

= $4,417,600 – $3,700,000 = $717,600

Over billing = Billed to date – Revenue to date

= $4,700,000 – $4,417,600 = $282,400

If over billing is a negative value, then it is called under billing. Hence,

       Under billing = Revenue to date – Billed to date

Over billing means that the construction company is borrowing money from the client by billing the latter an amount of revenue more than what the company has actually done. This does not mean that the total (final) revenue is to be billed wrongly, but is just the case that more money is received at an earlier stage of the work. On the contrary, under billing means that the company is allowing the client to borrow money from it, because it has incurred cost for doing work but without appropriately billed for the work.

There is an interesting point to note in the above example. The difference of the 'billed to date' and 'cost incurred to date' is $1,000,000 (i.e. $4,700,000 – $3,700,000). This $1,000,000 is the gross profit obtained based on the so called 'Straight Accrual Method'. (This method of calculating gross profit is rarely used in construction). The true gross profit (calculated by the Percentage of Completion Method shown just earlier) is $717,600, however. $1,000,000 – $717,600 = $282,400, and it can be seen that $282,400 is the over billing. In other words, the construction company billed the client for $4,700,000. It also incurred $3,700,000 as the 'cost of revenue', leaving it with $1,000,000 in hand. A part of this $1,000,000 is the true gross profit ($717,600), which belongs to the construction company, and the remaining part is over billing ($282,400), which belongs to the client.

A project can either be over billed or under billed and cannot be both. The above is an example of one project. A construction company usually has a number of projects in its hand at the same time. Some of the projects may be under billed and some may be over billed. Therefore, we can see that both under billing (current assets) and over billing (current liabilities) appear on the balance sheet.

1.6   Worked Examples

In this last section of Chapter 1, worked examples on the topics that have been discussed in this chapter are to be illustrated. There are eight short worked examples. It is hoped that readers will find them useful.

1.6.1   Worked Example 1

At the end of the fiscal year 2012, ABC Construction Co. has the following financial profile:

Total assets	$1,000,000
Total liabilities	$700,000
Gross profit	$250,000
Total operating expenses	$210,000

a) What is the total net worth of this company?

Net worth = Equity (Owner's capital) = Total assets – Total liabilities

= $1,000,000 – $700,000 = $300,000

b) What is the Net profit after tax? (Assume a 25% tax rate)

Net profit before tax = Gross profit – Total operating expenses

= $250,000 – $210,000 = $40,000

Net profit after tax = $40,000 – $10,000 = $30,000

1.6.2   Worked Example 2

The following balance sheet figures are of EFG Construction Company's:

	Current	Long-term	Total
Assets	$120,000	$110,000	$230,000
Liabilities	$100,000	$80,000	$180,000

Find the total net worth and working capital of the company.

Net worth (or Equity) = Total assets – Total liabilities

= \$230,000 – \$180,000 = \$50,000

Working capital = Current assets – Current liabities

= \$120,000 – \$100,000 = \$20,000

1.6.3   Worked Example 3

The following data regarding HJK Construction Company are:

Current assets	\$450,000
Current liabilities	\$350,000
Long-term liabilities	\$640,000
Net worth	\$340,000

a) What is the value of the company's fixed assets?

Net worth + Total liabilities = Total assets

\$340,000 + (\$350,000 + \$640,000) = (\$450,000 + Fixed assets)

Fixed assets = \$340,000 + \$350,000 + \$640,000 – \$450,000 = \$880,000

b) What are the working capital and current ratio?

Working capital = Current assets – Current liabilities

= $450,000 – $350,000 = $100,000

Current ratio = Current assets / Current liabilities

= $450,000 / $350,000 = 1.29

1.6.4   Worked Example 4

The founders of HJK construction company invested a total of $400,000 upon formation of the company two years ago. No dividends have been paid. Now the company has total assets of $170,000, current assets of $80,000, current liabilities of $95,000, and long-term liabilities of $50,000. Do you think that the company has been successful during its first two years of operations?

Before the start of the company, Equity = $400,000

After two years, Fixed assets = Total assets – Current assets = $170,000 – $80,000 = $90,000,

And, Equity = (Current Assets + Fixed Assets) – (Current Liabilities + Long term Liabilities)

= ($80,000 + $90,000) – ($95,000 + $50,000)

= $25,000

It shows that the company's Equity has decreased from $400,000 to $25,000 in two years which means a severe loss in these two years.

1.6.5   Worked Example 5

The working capital of LMN Construction Company at 31 Dec 20012 is $70,000. Current assets are $120,000, total assets are $250,000, and the company owes no long-term liabilities. What is the balance of owner's equity at 31 Dec 2012?

As we know: Working capital   =   Current assets – Current liabilities

$70,000   = $120,000 – X

X   =   Current liabilities = $50,000

And since: Owner's equity   =   Total assets – Total liabilities

= $250,000 – ($50,000 + 0) as long-term liabilities = 0

= $200,000

1.6.6   Worked Example 6

PQR Construction Co. expanded rapidly during 2012 by investing in new plant and equipment and other inventory. As a result, total assets increased from $400,000 at the beginning of 2012 to $700,000 at the end of 2012. The owner's equity at the beginning of the year was $250,000 and the company's net profit for the year was $50,000. Assuming the company paid no dividends during 2012,

a) what were the company's total liabilities at the beginning and at the end of the year if there is no change in capital stock of the company during 2012?

At the beginning of the year, Assets   =   Liabilities + Equity

$400,000   =   Y + $250,000

Total liabilities   =   Y = $150,000

At the end of the year, $700,000   =   Z + ($250,000 + $50,000)

Total Liabilities = Z = $700,000 – $300,000 = $400,000

b) what were the company's total liabilities at the end of 2012 if it received $200,000 in 2012 from the sale of additional capital stock?

Though the company received $200,000 during 2012 from the sale of additional capital stock, Equity as well as Assets increased in the same amount, and hence Liabilities remain the same which is $400,000.

As at 31 Dec 2012, Assets   =   Liabilities + Equity

i.e. ($700,000 + $200,000)   = $400,000 + ($300,000 + $200,000)

Or $900,000   = $400,000 + $500,000

1.6.7   Worked Example 7

The management of TUV Construction Company wishes to maintain a current ratio of 2 by reducing its short-term borrowing so as to achieve this result. The company's cash balance is currently $40,000. Accounts payable are equal to $55,000 and the company is now having a short-term borrowing of $35,000 from a bank. Inventory totals $95,000 and accounts receivable total $35,000. The current asset accounts consist of only cash, inventory, and accounts receivable; and the current liability accounts consist of only accounts payable and short-term bank loan payable. How much of its short-term bank loan should the company repay?

The problem can be presented by a table more clearly as follows:

Current Assets		Current Liabilities	
Cash	$40.000	Accounts Payable	$55,000
Inventory	$95,000	Short term bank loan	$35,000
Accounts Receivable	$35,000		

As the company wishes to maintain a current ratio of 2, it needs to pay back $10,000 to maintain such a ratio.

If the company uses $10,000 cash under current assets to pay back the short term bank loan under current liabilities, the short term bank loan will be $25,000 (i.e. $35,000 – $10,000).

Current liabilities = $55,000 + $25,000 = $80,000

Current assets = $30,000 + $95,000 + $35,000 = $160,000

Current ratio = Current assets / Current liabilities = $160,000 / $80,000 = 2

1.6.8   Worked Example 8

The following data are related to the operations of XYZ Construction Co. for 2012.

	Total contract sum	Total estimated costs	Cumulative costs incurred	Current year's costs	Cumulative contract billings
Completed contracts	$1,100,000	$900,000	$900,000	$360,000	$1,100,000
Contracts in progress	$5,400,000	$4, 500,000	$3,150,000	$1,850,000	$3,500,000

a) Determine the value of "costs and estimated earnings in excess of billings on works in progress" under the current asset account. (This is also called 'under billings')

Percentage of completion on contracts in progress
= (Cumulative costs incurred / Total estimated costs) × 100%
= ($3,150,000 / $4,500,000) × 100% = 70%
Revenue on contracts in progress = (Total contract sum) × (Percentage of completion)
= $5,400,000 × 70% = $3,780,000
Costs and estimated earnings in excess of billings on works in progress (or under billings)
= (Revenue on contracts in progress) – (Cumulative contract billings)
= $3,780,000 – $3,500,000 = $280,000

b) Compute the gross profit of the company applying the percentage-of-completion method.

Percentage of completion on current year's completed contracts
= ($360,000 / $900,000) × 100% = 40%
Revenue on current year's completed contracts
= $1,100,000 × 40% = $440,000
Percentage of completion on current year's contracts in progress
= ($1,850,000 / $4,500,000) × 100% = 41.11%
Revenue on current year's contracts in progress
= $5,400,000 × 41.11% = $2,220,000
Total revenue = $440,000 + $2,220,000 = $2,660,000
Total current year's costs incurred = $360,000 + $1,850,000 = $2,210,000
Gross profit of the company = $2,660,000 – $2,210,000 = $450,000

Exercise Questions for Chapter 1

Exercise Question 1

Using the company balance sheet shown on Table 2.2 of Chapter 2, calculate for each 2012 and 2011:

    a) the company's equity (or net worth),
    b) working capital, and
    c) current ratio.

Exercise Question 2

Based on the project data presented in the table below, calculate for each of the two projects:

    a) the revenue using the percentage-of-completion method,
    b) the gross profit to date, using the percentage-of-completion method, and
    c) the amount of over / under billing for each project.

Project Financial data	Project A	Project B
Contract amount	$15,000,000	$15,000,000
Original estimated cost	14,400,000	14,800,000
Amount billed to date	10,700,000	10,700,000
Payments received to date	10,900,000	10,630,000
Cost incurred to date	11,450,000	10,550,000
Forecasted cost to complete	3,000,000	4,100,000
Costs paid to date	9,400,000	9,600,000

Note: some figures are for reference only and are not useful for calculating what are asked for.

2   Financial Statement Analysis of a Construction Company

2.1   Analysis of Financial Statements by Financial Ratios

In financial statement analysis, Financial Ratios help a lot in indicating the financial health of a construction company. In this chapter, we are going to discuss these ratios first, and then to have a case study to see how we may use them to help management decisions.

The financial ratios relevant to the construction industry can be classified into five categories. They are (1) Profitability Ratios, (2) Liquidity Ratios, (3) Working Capital Ratios, (4) Capital Structure Ratios, and (5) Activity Ratios. These ratios will be discussed in detail in Section 2.2.

2.2   Five Categories of Financial Ratios

In this section, we have to refer to Tables 1.1 and 1.2 of Chapter 1. The explanation of the financial ratios below will be based on these two tables – Income Statement and Balance Sheet (for 2012 only) – shown in Chapter 1.
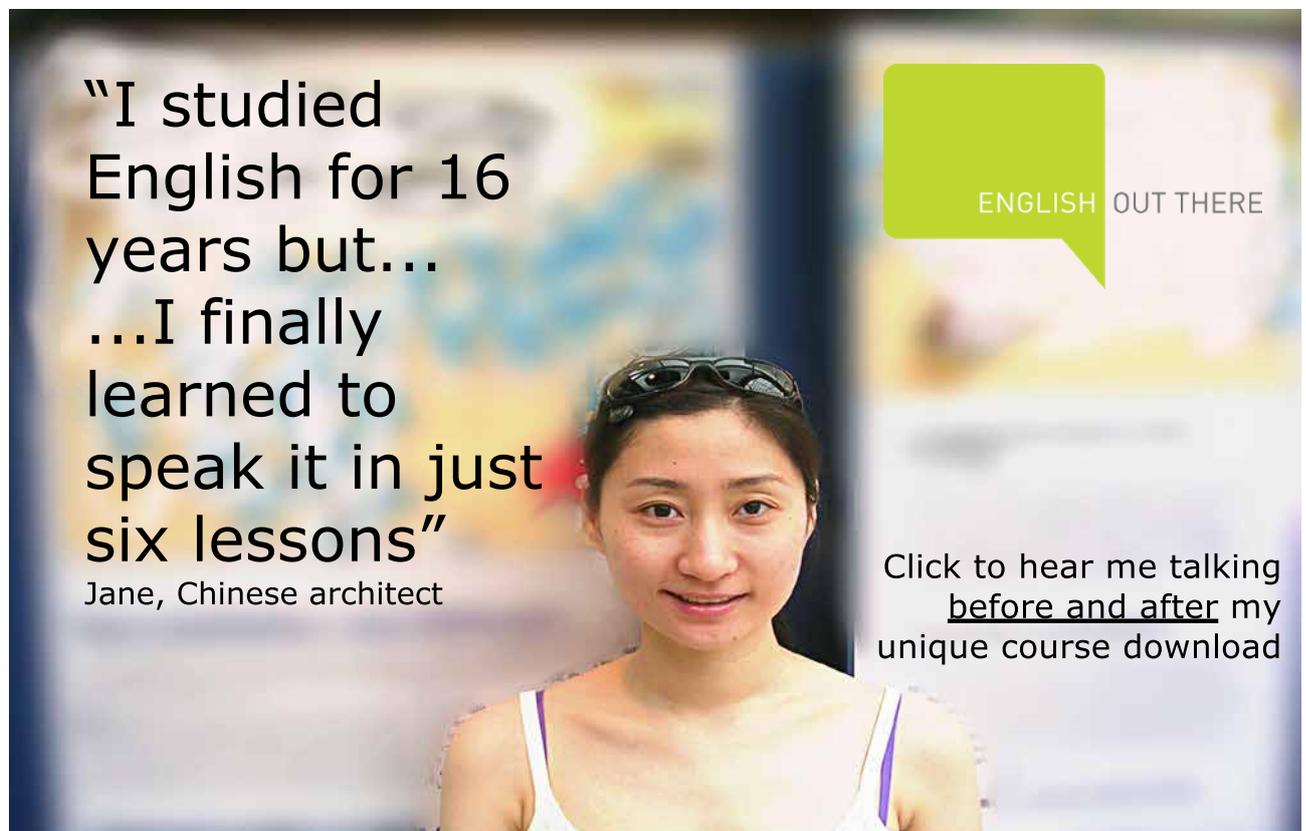
2.2.1   Profitability Ratios

Profitability ratios measure the construction company's ability to earn profit from its operation. The three most commonly used profitability ratios are:

Gross Profit Margin Ratio   = Gross profit / Revenue
= 8,098,000 / 40,185,000 = 20.15%

(The goal for net profit margin ratio is 25% minimum; if subcontractors (pay-as-paid basis) occupy a significant portion of the cost of revenue, the goal can be reduced to 20% minimum)

Net Profit Margin Ratio   = Net profit before tax / Revenue
= 2,704,000 / 40,185,000 = 6.73%

(The goal for net profit margin ratio is 5% minimum)

Return on Equity Ratio   = Net profit before tax / Owners' equity
= 2,704,000 / 8,412,675 = 32.14%

(The return on equity ratio should be between 15% and 40%)

2.2.2   Liquidity Ratios

Liquidity ratios indicate the construction company's ability to pay its obligations as they come due. The three most common liquidity ratios used are shown below.

Current Ratio = Current assets / Current liabilities
= 12,289,500 / 8,450,835 = 1.45

(The current ratio should be higher than 1.3 for a financially healthy construction company)

Acid Test Ratio (or **Quick Ratio**) = (Cash + Accounts receivables) / Current liabilities
= (2,589,000 + 5,767,000) / 8,450,835 = 0.99

(The acid test ratio or quick ratio should be higher than 1.1 for a construction company)

Current Assets to Total Assets Ratio = Current assets / Total assets
= 12,289,500 / 20,392,067 = 60.27%

(The current assets to total assets ratio should be between 60% and 80%)

2.2.3   Working Capital Ratios

These ratios measure how well the construction company is utilizing its working capital. The three most commonly used working capital ratios are shown below.

Working Capital Turnover = Revenue / Working capital
= 40,185,000 / (12,289,500 − 8,450,835) = 10.47 times

(The working capital turnover should be between 8 and 12 times per year)

Net Profit to Working Capital Ratio = Net profit before tax / Working capital
= 2,704,000 / (12,289,500 − 8,450,835) = 70.44%

(The net profit to working capital ratio should be between 40% and 60%)

Degree of Fixed Asset Newness = Net depreciable fixed assets / Total depreciable fixed assets
= 8,102,567 / 20,631,940 = 39.27%

(The degree of fixed asset newness should be between 40% and 60%)

2.2.4   Capital Structure Ratios

Capital structure ratios indicate the ability of the construction company to manage liabilities. These ratios also indicate the approach that the company prefers to finance its operation. The two major capital structure ratios are:

Debt to Equity Ratio = Total liabilities / Owners' equity
= 11,979,392 / 8,412,675 = 1.42

(The debt to equity ratio should be lower than 2.5)

Leverage   = Total assets / Owners' equity
= 20,392,067 / 8,412,675 = 2.42

Or

Leverage   = Total assets / Owners' equity
= (Total liabilities + Owners' equity) / Owners' equity
= (Total liabilities / Owners equity) + 1
= Debt to Equity Ratio + 1
= 1.42 + 1 = 2.42

(The leverage should be lower than 3.5. Some construction companies prefer to use leverage of 3.5 or close to it but some conservative ones prefer to use a lower leverage. This relates to, of course, the use of a higher or lower debt to equity ratio by the company.)

2.2.5 Activity Ratios

Activity ratios indicate whether or not the construction company is using its assets effectively, and if yes, how effective they are. There are quite a number of activity ratios, and the seven commonly used ones are shown below.

Average Age of Material Inventory = (Material inventory / Materials cost) × 365 days
$$= (850,000 / 13,000,000) \times 365 = 23.87 \text{ days}$$

(The average age of material inventory should be shorter than 30 days)

Average Age of Under Billings = (Under billings / Revenue) × 365 d
$$= (547,250 / 40,185,000) \times 365 = 4.97 \text{ days}$$

(The average age of under billings should be the shorter the better)

Average Age of Accounts Receivable = (Accounts receivable / Revenue) × 365 d

= (5,767,000 / 40,185,000) × 365 = 52.38 days

(The average age of accounts receivable should be shorter than 45 days)

Cash Conversion Period = Average age of material inventory + Average age of under billings

+ Average age of accounts receivable

= 23.87 + 4.97 + 52.38 = 81.22 days

(The cash conversion period should be shorter than 75 days)

Average Age of Accounts Payable = [Accounts payable / (Materials + Subcontracts)] × 365 d

= [4,325,250 / (13,000,000 + 12,500,000)] × 365

= 61.91 days

(The average age of accounts payable should be shorter than 45 days)

Average Age of Over Billings = (Over billings / Revenue) × 365 d

= (617,205 / 40,185,0 2 gl00) × 365 = 5.61 days

(Usually there is no guideline on average age of over billings)

Cash Demand Period = Cash conversion period – Average age of accounts payable

– Average age of over-billings

= 81.22 – 61.91 – 5.61 = 13.70 days

(The cash demand period should be shorter than 30 days)

Financial statement analysis involves the analyses of all the above ratios. We are going to have a case study in Section 2.3. We will see how we can use these ratios to analyze the financial health of a construction company by reviewing its income statement and balance sheet.

2.2   A Case Study

First of all, the income statement and the balance sheet of the construction company under study are shown in Tables 2.1 and 2.2 respectively. The general manager of the company recently resigned. A new one has just been hired. The new general manager is reviewing the financial statements and hopes to find out the goods and the bads of the company and hence find ways to make improvement.

1. At the very first glance of the Income Statement, the new general manager notices that the revenue has increased by 6,174,101 from 34,701,250 in 2011 to 40,875,351 in 2012. The increment is about 17.79%. So, he is wondering if it is a very positive sign. After second thought, he reminds himself that this is only a preliminary analysis. Although it is correct to look at the trend of the revenue as a first step, he can only say that it might be a good sign but is too early to say that it is a very positive sign. Perhaps it is too early to be too happy about it. Further investigation on the financial statements is certainly necessary.

Income Statement

	31/12/2012	31/12/2011	Vertical Analysis % 31/12/2012	Vertical Analysis % 31/12/2011	Horizontal Analysis $ 31/12/2012	Horizontal Analysis %
Revenue	40,875,351	34,701,250	100.00%	100.00%	6,174,101	17.79%
Cost of Revenue						
Materials	20,732,506	15,925,567	50.72%	45.89%	4,806,939	30.18%
Labour	3,317,123	3,307,879	8.12%	9.53%	9,244	0.28%
Subcontracts	6,417,407	4,721,312	15.70%	13.61%	1,696,095	35.92%
Other direct costs	487,059	426,885	1.19%	1.23%	60,174	14.10%
Total Cost of Revenue	30,954,095	24,381,644	75.73%	70.26%	6,572,452	26.96%
Gross Profit	9,921,256	10,319,606	24.27%	29.74%	(398,350)	-3.86%
Operating Expenses						
Variable overhead						
Advertising	898,398	826,927	2.20%	2.38%	71,471	8.64%
Truck, vehicle and equipment	1,164,800	864,500	2.85%	2.49%	300,300	34.74%
Insurance, workers' compensation	429,958	319,058	1.05%	0.92%	110,900	34.76%
Travel and entertainment	450,893	184,606	1.10%	0.53%	266,287	144.25%
Bad debts	165,935	98,706	0.41%	0.28%	67,229	68.11%
Miscellaneous	42,985	46,197	0.11%	0.13%	(3,212)	-6.95%
Total Variable overhead	3,152,968	2,339,994	7.71%	6.74%	812,975	34.74%
Fixed overhead						
Depreciation	555,220	560,500	1.36%	1.62%	(5,280)	-0.94%
Rent	165,000	140,000	0.40%	0.40%	25,000	17.86%
Salaries	3,760,500	3,555,200	9.20%	10.25%	205,300	5.77%
Legal and audit	101,997	83,542	0.25%	0.24%	18,455	22.09%
Maintenance and repair	176,598	96,960	0.43%	0.28%	79,638	82.13%
Office Supplies	158,221	132,982	0.39%	0.38%	25,239	18.98%

Utilities	210,950	135,800	0.52%	0.39%	75,150	55.34%
Miscellaneous	54,872	50,699	0.13%	0.15%	4,173	8.23%
Total Fixed overhead	5,183,358	4,755,683	12.68%	13.70%	427,675	8.99%
Total Operating Expenses	8,336,326	7,095,677	20.39%	20.45%	1,240,650	17.48%
Operating Profit	1,584,930	3,223,930	3.88%	9.29%	(1,639,000)	-50.84%
Other Income/Expense						
Interest income	46,000	37,550	0.11%	0.11%	8,450	22.50%
Interest expense	(297,490)	(446,750)	0.73%	1.29%	(149,260)	-33.41%
Net Profit before Tax	1,333,440	2,814,730	3.26%	8.11%	(1,481,290)	-52.63%
Tax Expense (25% tax rate)	333,360	703,682				
Net Profit after Tax	1,000,080	2,111,047	2.45%	6.08%	(1,110,967)	-52.63%

Table 2.1 – Income Statement for the Case Study

Note:

Vertical analysis – the percentages of all items in the income statement are calculated based on the Revenue of the same year (either 2011 or 2012)

Horizontal analysis – a comparison between the two years (2011 and 2012)

Balance Sheet		
	31/12/2012	31/12/2011
Assets		
Current Assets		
Cash	2,305,078	1,877,676
Accounts receivable	6,124,992	5,837,658
Retention money	1,743,663	1,659,415
Material Inventory	942,765	761,763
Costs and estimated earnings in excess of billings on work in progress	581,221	486,472
Prepaid expenses and others	1,000,026	1,062,968
Total Current Assets	12,697,745	11,685,952
Fixed Assets		
Property and equipment	2,655,000	2,580,000
Construction plant	806,200	800,000
Vehicles/Trucks	414,500	310,000
Furniture and fixtures	69,560	60,100
Total depreciable assets	3,945,260	3,750,100
Less accumulated depreciation	2,051,900	1,663,400
Net Fixed Assets	1,893,360	2,086,700
Total Assets	14,591,105	13,772,652
Liabilities		
Current Liabilities		
Accounts payable	3,930,309	3,481,330
Accrued expenses	1,441,215	1,076,450
Notes payable	588,149	358,817
Retention money payable	835,495	551,763
Billings in excess of costs and estimated earnings on work in progress	560,847	495,167
Other current liabilities	323,232	213,478
Total Current Liabilities	7,679,247	6,177,005
Long-term Liabilities	1,480,513	1,901,445
Total Liabilities	9,159,760	8,078,450
Equity (i.e. Net Worth)		
Capital stock	2,000,000	2,000,000
Additional paid-in capital	800,000	800,000
Retained earnings	2,631,345	2,894,202
Total Equity	5,431,345	5,694,202
Equity + Total Liabilities	14,591,105	13,772,652

Table 2.2 – Balance Sheet for the Case Study

2. Very soon, the new general manger looks into the trend of the gross profit margin of the company. He finds that the gross profit margin ratios are 24.27% and 29.74% in 2012 and 2011 respectively, representing a fall of 5.47%. He should be cautious about this fall as a fall in this ratio means less profit will be earned for every dollar of revenue generated. Also, the gross profit falls by 3.86% even though there is a rise of 17.79% in the revenue. It may be due to an increase in the cost of revenue without any increase in the contract price because of the highly competitive market at that moment, or unfavourable purchasing/markup policies, and so on. The general manager therefore should make a further investigation on the cost of revenue, that is, the costs of materials, labour, subcontracts, etc.

3. All items in the cost of revenue have a 2-digit increment in percentage except the labour cost, whose increase is insignificant (up by 0.28%). The general manager is even more impressive to find out that the labour cost as a percentage of the revenue has actually gone down (see vertical analysis of the income statement) while the revenue has gone up. One thing he needs to take note of is that the cost of subcontractors has risen by 35.92% compared to the previous year (see horizontal analysis). The reduction in labour cost may be due to the increase in subcontracts in 2012. Anyway, the general manager needs to find out whether or not the reduction of labour has been really due to the increase of subcontracts as they are somehow interrelated. Keeping a certain level of direct labour in the company is essential and a balance has to be maintained between the company's direct labour and subcontractors.

4. Upon further review of the Income Statement, the new general manager is rather disappointed by the performance of 2012 as there is a significant decrease of 52.63% in the net profit although the revenue has gone up. He requests a thorough study on the reasons for the decrease and a report from the management. He strongly thinks that the study is necessary and important because of such a discouraging performance. In fact, the net profit margin ratio is only 3.26% (see vertical analysis), which is well below the norm of the construction industry's minimum 5%. He is worrying about the company's ability to achieve a satisfactory return on its investment.

The rise of total variable overhead is 34.74% from 2011 to 2012 (see horizontal analysis). This rise certainly lowers down the operating profit considerably. When he looks into the items of the variable overhead, he is troubled by the acute rise of almost 2.5 times in the Travel and Entertainment cost. The increase of Bad Debts by 68.11% is also high. Both Truck, Vehicle & Equipment expenses and Insurance & Workers' Compensation expenses have risen by almost 35%. They are not good signs as well. A thorough investigation on "how? why? what?" of these expenses is also necessary. Why there is a big increase in the entertainment expenses? What are the guidelines in approving entertainment expenses? How is the collection policy on debtors? Does the Accounts Department follow the policy strictly? What kinds of debtors are they?

The total fixed overhead as 12.68% of the revenue in 2012 is a little down compared to 2011 (see vertical analysis). Also, the increment is 8.99% in 2012 compared to 2011 (see horizontal analysis). Overall speaking, it is quite acceptable. However, the general manager needs to pay special attention to the sharp increases in Maintenance & Repair expenses of 82.13% and Utilities expenses of 55.34%.

5. While the general manger knows that the net profit margin ratio is lower than the industry's norm, he also sees that 24.55% of a return on equity ratio is a satisfactory return in the industry. It is quite clear that the operation of the business has delivered a good return to the owners (partners or shareholders). Therefore, what he really needs to do now are those that have been said in Point 4. Doing them should be sufficient at the time being.

6. The general manager notes that the Maintenance and Repair cost in 2012 is much higher than that in 2011. This can be a signal to replace aging assets. The degree of fixed asset newness is found to be 47.99%. In fact, the overall indicator is healthy. Probably there are some particularly old assets among the fixed assets. In any case, the general manager needs to look closely into this matter so as to identify if there are really such old assets, and if yes, he should replace them in order to save repair costs. Furthermore, the high Maintenance and Repair cost may be due to poor management in cost control in that particular department which he should be responsible to rectify.

Click on the ad to read more

Download free eBooks at bookboon.com

7. The general manager observes the declined working capital in 2012. However, he is not worrying about the company's ability to meet its short-term obligations in the coming year (2013), because he thinks the current ratio of 1.65 (for 2012) is a very good one. On the top of that, the working capital turnover is 8.14 times. It has improved as compared to 2011. Actually, 8.14 turns per year is a favourable level. Provided that the current ratio is healthy, if he can minimize the working capital so that fewer funds are tied up, he will be able to maximize cash flows,. Hence, the available cash can then be more profitably invested in the business.

8. Upon further review, the general manager finds that the quick ratio or acid test ratio is 1.10 (just good enough according to the norm of the construction industry) for 2012. The general manager is convinced that the company is financially secure in short term obligations. The quick ratio is in fact a more rigorous assessment of a company's short term financial ability than the current ratio. However, he cannot rely solely on the quick ratio itself as it provides no information about the timing of cash flows. The timing is crucial too as it really determines the ability of a company to pay debts when due. So it should be more meaningful to consider the quick ratio together with the cash demand period (Cash Demand Period is also called Cash Conversion Cycle – in days). In this case study, the cash demand period is 18.63 days, which is considered very satisfactory. Therefore, the company's short term financial ability is good.

9. The general manager is interested to know how well the company can handle unpredictable future cash flows. A quick way to the answer is by looking at the debt to equity ratio of the company. For 2012 the ratio is 1.69, which is less than 2.5. The figure 1.69 is a healthy but rather low debt to equity ratio in some people's eyes in the construction industry. It means that the borrowing capability of the company is strong to meet unpredictable expenses or to service its unexpected long term liability in the future. To some people, however, such a rather low debt to equity ratio cannot allow the company to earn a larger money with a limited equity (see Section 4.4 of Chapter 4). The leverage is only 2.69, whereas the norm of the industry's maximum can be 3.5. Yet, the general manager is a rather conservative man. He is satisfied with the debt to equity ratio being 1.69 and the leverage being 2.69. He is a debt averter, at least so at the time being when he is so new in the company. Even if he wants to take a bigger risk, it would not be now but would be in the future. He actually knows well the drawbacks of high debts.

Drawbacks of high debts:

a) Low ability to handle unpredictable expenses.
b) Low ability to service unexpected long term liability.
c) Low grade of credit rating.
d) High interest expenses.
e) Sureties believe that construction companies with higher levels of working capital (i.e. lower current liabilities or lower debts) have a better chance of successfully finishing their projects.
f) Difficult to survive in economic downturn.
g) High risk in insolvency.

h) In general, the more debt a company has, the riskier its stockholder is, since debt holders have first claim to a company's assets. So if a company becomes bankrupt, there may be very little left to its stockholders after the company has satisfied its debt holders.

10. The general manager worries about the long period of average age of accounts receivable (54.69 days is longer than the industry's norm 45 days). He requests the management to have a better control on this aspect. His request is understandable especially when there is a sharp rise of 68% of bad debts in 2012 (see Point 4). In general, the greater the number of days outstanding, the greater the probability of delinquencies in accounts receivable. Also, the company is indirectly extending interest-free loans to its clients by maintaining accounts receivable. Therefore, it is correct for the general manager to ask the management to review its credit policies in order to ensure the timely collection of imparted credit.

11. The general manager is delighted with the efficacy of the inventory management as the average age of material inventory is 16 days for 2012 and 17 days for 2011, which are well within the industry's norm – a maximum of 30 days. An efficient manager has to balance the fund tied up in inventory as well as meeting the needs of the projects in hand. For on the other hand, over aggressiveness in reducing the age of inventory may cause out of stock issues.

12. Although the average age of accounts payable is higher than the normal practice 45 days, the general manager is not concerned about it at all. His philosophy is that the longer the payable days, the company will have more cash available in hand. However, on the other hand, he should not ignore building good relationship with his vendors. In fact, he needs the support and contribution from vendors from time to time to run his business efficiently and effectively. Moreover, the company can earn cash discount on timely payment as well as more favorable buying contracts from vendors, and there are both tangible and intangible benefits.

Exercise Questions for Chapter 2

Exercise Question 1

Base on the Income Statement and the Balance Sheet shown on Tables 2.1 and 2.2 respectively in Chapter 2. Calculate:

a) the three Profitability Ratios,
b) the three Liquidity Ratios,
c) the three Working Capital Ratios,
d) the two Capital Structure Ratios, and
e) the seven Activity Ratios.

Exercise Question 2

By referring to the ratios calculated in Exercise Question 1 above, are there any things you would like to add to Section 2.2 of the chapter to remind the new general manager that he has missed but should have considered?

3 Compound Interest, Net Present Value (NPV), Equivalent Annual Cost and Loan Redemption

3.1 Nominal Interest Rate and Real Interest Rate

If an amount of $100,000 is put in a bank, and the bank offers an interest rate of 6% per annum compounded annually, we will get $133,822.56 after 5 years. This is calculated by:

$$\$100,000 \times (1 + 0.06)^5 = \$133,822.56$$

In general, the compound amount is calculated by the formula

$$S = P \times (1 + i)^n \qquad \text{(Eq 3.1)}$$

where P = the initial investment or the principal sum

n = the number of periods, which refers to years or months, or even days

i = interest rate in % per period

S = the total sum of compound amount accumulated after n periods

The bank interest rate 6% per year (or per annum) is also called "nominal rate" (or "apparent rate"). The term "nominal" means that the rate is not a "real rate". In other words, the real value of the initial sum $100,000 does not grow at 6% per annum, but usually grows at a rate lower than 6% per annum because inflation usually exists. If we assume the inflation rate to be f % per annum, the nominal rate to be i' % per annum, and the real rate to be i % per annum, then we have the following approximate relationship:

$$i' = i + f \qquad \qquad \text{(Eq 3.2)}$$

It is an approximate relationship only and is nearly true if the values of i and f are small. The true relationship will be discussed in Chapter 5. If the inflation rate f in a year is very high, say, higher than the bank's interest rate i' (nominal rate or apparent rate), then we will have a negative real interest rate. This means that if we put money into a bank we will in fact lose money, because the value of the accumulated amount we get is lower than the value of the principal amount we initially put. We must not forget one thing, that is, the bank's interest rate is always a nominal rate, never a real rate. Investors, however, are only interested in the real rate, because only the real rate can reflect how much they really earn.

Now, let us take a look at an example how banks pay interests to their customers.

Example 3.1

a) A bank pays 6% interest per annum, but interest payment is on yearly basis. If $100,000 is deposited on 1 January, providing no withdrawal is made, how much will be in the account on 31 December of the same year?
The accumulated amount = $100000 \times (1 + \frac{0.06}{1}) = \$106,000$

b) The bank keeps paying 6% interest per annum, but interest payment is on half yearly basis. How much will be in the account at the year end?
The accumulated amount = $100000 \times (1 + \frac{0.06}{2})^2 = \$106,090$

c) The bank keeps paying 6% interest per annum, but interest payment is on quarterly basis. How much will be in the account at the year end?

The accumulated amount = $100000 \times (1 + \frac{0.06}{4})^4 = \$106,136$

d) If the bank keeps paying 6% interest per annum, how much will be in the account at the year end if it pays interest on monthly basis?
The accumulated amount = $100000 \times (1 + \frac{0.06}{12})^{12} = \$106,168$

e) If the bank keeps paying 6% interest per annum, how much will be in the account at the year end if it pays interest on daily basis?

The accumulated amount $= 100000 \times (1 + \frac{0.06}{365})^{365} = \$106,183$

f) If the bank offers to pay interest (at 6% per annum) as often as the customer chooses, what will be the maximum amount in the account at the end of the year?

We want to find $100000 \times (1 + \frac{0.06}{n})^{n}$ for $n \to \infty$.

Since $\lim_{n\to\infty}(1 + \frac{x}{n})^{n} = e^{x}$, the accumulated amount $= 100000 \times e^{0.06} = \$106,184$

3.2 Compound Interest Calculations

In the discussion of compound interest mathematics, we want to make an assumption – an inflation-free assumption, for the time being. We assume that there is no inflation in this world, and hence i is equal to i'. This assumption is of course not true in the real world. The purpose of making such an assumption temporarily is that we do not want to create confusion between nominal rate and real rate at the time being. We will remove this unrealistic assumption later after the next chapter and go back to live in the real world.

3.2.1 Uniform Series Compound Amount

In section 3.1, we have only considered a principal sum of single payment. Now we begin to see a series of uniform payments. For a series of uniform payments, let A = the periodic uniform payment made at the end of each period continuing for n periods to accumulate a sum S. The situation can be presented diagrammatically as shown in Fig. 3.1.
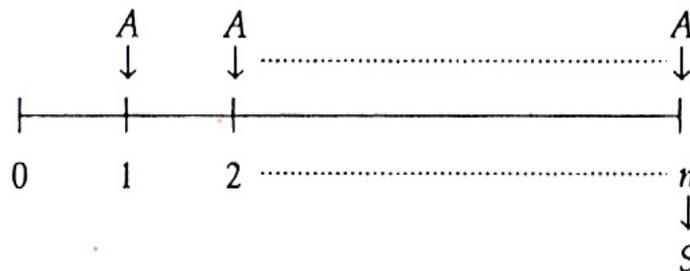
Fig. 3.1 – Sum of money accumulated after n periods due to uniform periodic payments.

Compound amount of the first A at period n	$= A(1 + i)^{n-1}$
Compound amount of the second A at period n	$= A(1 + i)^{n-2}$
:	:
Compound amount of the $(n-1)$th A at period n	$= A(1 + i)^{1}$
Compound amount of the nth A at period n	$= A$
Total	$= S$

So it is a geometric series. We can apply geometric series formula to obtain that

$$S = A \times \left[\frac{(1+i)^n - 1}{i} \right] \qquad \text{(Eq 3.3)}$$

The values of $\left[\dfrac{(1+i)^n - 1}{i} \right]$ are shown in the Appendix for different values of i and n.

Example 3.2

A construction company will replace an excavator after 5 years. A new one costs $250,000. How much is the end-of-year annual uniform payment the company has to put into a bank in order to save enough money in five years' time for purchasing the equipment if the bank is offering an interest rate of 4% per annum?

The problem can be presented diagrammatically as follows:

Fig. 3.2 – Sum of $250,000 accumulated by 5 uniform periodic (annual) payments.

We have to bear in mind that the excavator always costs $250,000, whether now or after five years, as the inflation-free assumption has been made.

Applying Equation 3.3, $\quad 250{,}000 = A \times \left[\dfrac{(1+i)^n - 1}{i} \right] = A \times 5.4163$

(5.4163 is found by substituting $i = 0.04$ and $n = 5$ into the formula, or from Appendix)

$$\text{Hence, } A = \frac{250{,}000}{5.4163} = \$46{,}157$$

3.2.2   Uniform Series Sinking Fund

It is a little troublesome to obtain $46,157 in Example 3.2. First, we have to use Equation 3.3. Second, we have to rearrange terms to calculate A (A is usually referred to as Sinking Fund). In fact, we can calculate the sinking fund A in a more direct (quicker) way.

Equation 3.3 can be rearranged as follows:

$$A = S \times \left[\frac{i}{(1+i)^n - 1} \right] \qquad\qquad\qquad\qquad (\text{Eq 3.4})$$

Equation 3.4 allows us to calculate the sinking fund $46,157 (i.e. A) in Example 3.2 directly and more quickly. It is left to the readers as an exercise to try it.

The values of $\left[\dfrac{i}{(1+i)^n - 1} \right]$ are shown in the Appendix for different values of i and n.

3.2.3 Uniform Series Capital Recovery

In the previous two subsections, the relationships between the final accumulated sum and a principal sum investment or a series of uniform installment investment are discussed. This subsection discusses the relationship between an initial loan *P* and the subsequent uniform series of payments to offset against the loan *P*. The situation can be presented in Fig. 3.3 diagrammatically.
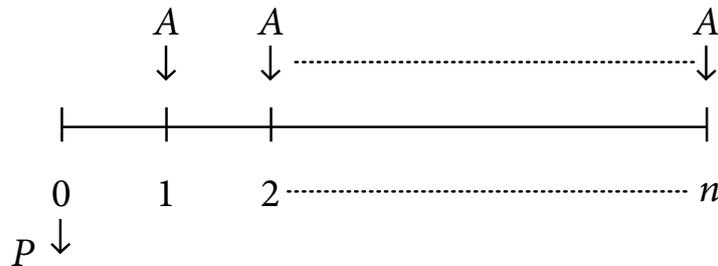
Fig. 3.3 – Uniform periodic payments for recovering a loan *P*

Since Equation 3.1 gives $S = P \times (1 + i)^n$

and Equation 3.3 gives $S = A \times \left[\dfrac{(1+i)^n - 1}{i} \right]$

Solving the above two equations by eliminating *S*, we obtain

$$A = P \times \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right] \qquad\qquad \text{(Eq 3.5)}$$

The values of $\left[\dfrac{i(1+i)^n}{(1+i)^n - 1} \right]$ are shown in the Appendix for different values of *i* and *n*.

We are going to see an example how we can apply Equation 3.5.

Example 3.3

If the excavator of Example 3.2 suddenly breaks down now so the company needs to purchase a new one immediately and not 5 years later, and therefore has to borrow from a bank $250,000 at an interest rate of 4% per annum. What will be the annual installment for repaying the loan in 5 years?

The problem can be presented diagrammatically as follows:

$$
\begin{array}{cccccc}
 & A & A & A & A & A \\
 & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
\vdash & \!\!+\!\! & \!\!+\!\! & \!\!+\!\! & \!\!+\!\! & \dashv \\
0 & 1 & 2 & 3 & 4 & 5
\end{array}
$$

$\downarrow$
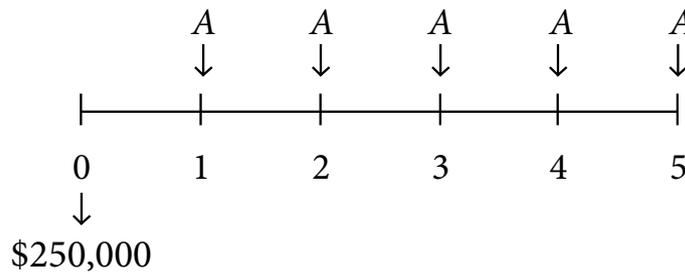
$250{,}000

Fig. 3.4 – Uniform annual installments for recovering a loan of $250,000.

$$
A = 250{,}000 \times \left[\frac{i\,(i)^n}{(i)^n} \right] = 250{,}000 \times (0.22463)
$$

$$
= \$56{,}157
$$

(0.22463 is found by substituting $i = 0.04$ and $n = 5$ into the formula, or from Appendix)

It should be interesting to note that the answer of Example 3.3, which is $56,157, is exactly $10,000 more than the answer of Example 3.2, which is $46,157. This is not a coincidence. The later part of this chapter will give hints to the explanation. Readers may know why it is so after they have finished reading Example 3.6, particularly the last part of it.

The value A in Example 3.3 is usually referred to as Equivalent Annual Cost as the period is defined as one year in the question. For example, the equivalent annual cost of owning the excavator in Example 3.3 is \$56,157 a year with i equal to 4% per annum. Comparatively, the annual cost of owning it is \$50,000 a year (i.e. 250,000 ÷ 5) if i is taken as 0% p.a. (i.e. zero interest rate; p.a. stands for per annum).

3.2.4 Present Value (Single payment and Uniform Series Payments)

Let us look at Fig. 3.5, which explains what Present Value is for a single payment.

$$\$10,000 \qquad\qquad F = \$10,000 \times (1+i)^{10}$$
$$\downarrow \qquad\qquad\qquad \downarrow = \$16,289$$

$$0 \quad 1 \quad 2 \text{-----------------------} 10 \text{ year}$$
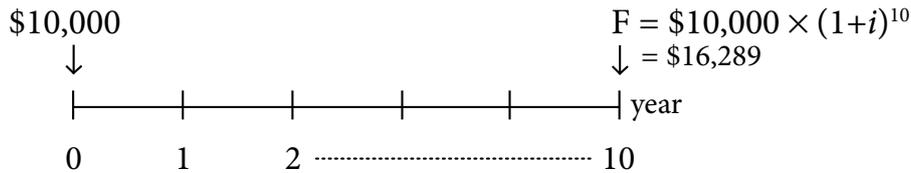
Fig. 3.5 – Present value of a single payment F.

F would be equal to \$16,289 if i is taken as 5% p.a. In other words, \$16,289 of 10 years later is equivalent to \$10,000 today, or we can say that the present value (i.e. today's value) of \$16,289 of ten years later is \$10,000, because:

$$\$10,000 \times (1 + 0.05)^{10} = \$16,289$$

Mathematically, the present value of F can be expressed in the form of:

$$\text{Present value of } F = \frac{F}{(1+i)^{n}} = F \times \left[\frac{1}{(1+i)^{n}} \right] \qquad \text{(Eq 3.6)}$$

The value of $\left[\dfrac{1}{(1+i)^{n}} \right]$ are shown in the Appendix for different values of i and n.

Now, let us consider a uniform series of payments as shown in Fig. 3.6:

$$F \qquad F \qquad\qquad\qquad F$$
$$\downarrow \qquad \downarrow \text{-----------------------} \downarrow$$

$$0 \quad 1 \quad 2 \text{-----------------------} n$$
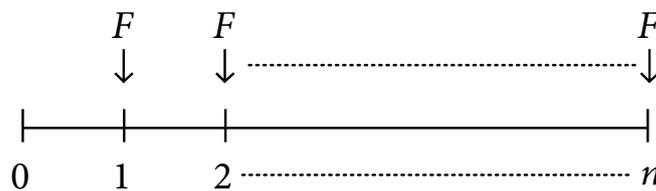
Fig. 3.6 – Present value of uniform periodic payments.

The equivalent sum of money *today* (the present value), which is equivalent to all these uniform series of payment for n periods, can be calculated from the equation:

$$\frac{F}{1+i} + \frac{F}{(1+i)^{2}} + \frac{F}{(1+i)^{3}} + \dots + \frac{F}{(1+i)^{n}} = F \times \left[\frac{(1+i)^{n}}{i(1+i)^{n}} \right] \qquad \text{(Eq 3.7)}$$

The above equation is just a simple geometric series and its derivation is left to the readers.

The value of $\left[\dfrac{(1+i)^n - 1}{i(1+i)^n} \right]$ are shown in the Appendix for different values of i and n.

Now, let us see an example that the application of present value is illustrated.

Example 3.4

The construction cost of a service reservoir for supplying water to a housing estate is estimated to be $60,000,000. The annual operation and maintenance cost is estimated to be $2,500,000 per year. The annual income from the collection of water supply fees (for this service reservoir's part) from the users will be $7,000,000. Assuming a time horizon of 30 years and taking i as 5% p.a., find out if the project is financially feasible.

The problem can be presented diagrammatically in Fig. 3.7 as follows:
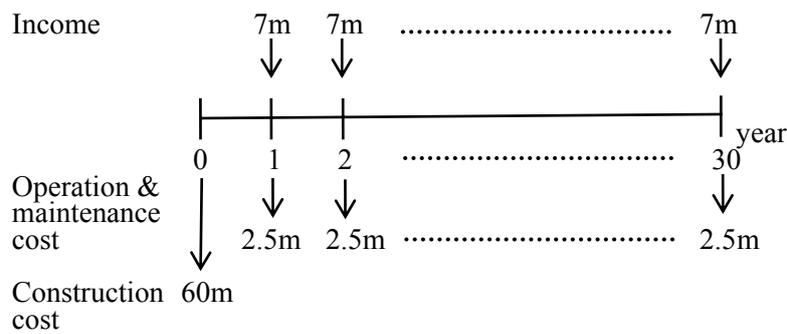
Fig. 3.7 – Cash flows presented in diagram for service reservoir project

a) **Present Value Method**

  (i)   Present value of benefits in 30 years

$$= 7,000,000 \times \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right] \qquad \text{for } i = 0.05 \text{ and } n = 30$$

$$= 7,000,000 \times (15.3724) = \qquad\qquad\qquad\qquad \$107,606,800$$

  (ii)   Present value of construction cost = $60,000,000

        Present value of operation and maintenance cost

$$= 2,500,000 \times \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right] \text{ for } i = 0.05 \text{ and } n = 30$$

$$= 2,500,000 \times (15.3724) = \qquad\qquad \underline{\$38,431,000} \quad \underline{\$98,431,000}$$

  (iii) Net present value (NPV) = $9,175,800

The positive net present value (i.e. present value of total benefits – present value of total costs) indicates that the project is feasible. NPV is the short form of net present value.

b) **Equivalent Annual Cost Method**

(i) Annual benefit = $7,000,000

(ii) Annual operation and maintenance cost = $2,500,000

Equivalent annual cost of construction

$$= 60,000,000 \times \left[\frac{i(i^n)}{(i^n)} \right] \text{ for } i = 0.05 \text{ and } n = 30$$

$= 60,000,000 \times (0.06505) =$ $\underline{\$3,903,000}$ $\underline{\$6,403,000}$

(iii) Net annual benefit (NAB) = $597,000

The positive net annual benefit (i.e. total annual benefits – total annual costs) indicates that the project is feasible. NAB is the short form of net annual benefit.

The two methods (a) and (b) must have consistent results. If the NPV is calculated to be positive, the NAB must also be calculated to be positive, and vice versa. Therefore, either one method only is sufficient to determine whether the project is feasible or not.

We should note that there exists a relationship between the results of methods (a) and (b):

$$\text{i.e.} \quad \$9,175,800 \ \times \ \left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] \ = \ \$597,000 \qquad (\text{for } i = 0.05 \text{ and } n = 30)$$

$$\underset{\text{(NPV)}}{\uparrow} \qquad\qquad\qquad\qquad \underset{\text{(NAB)}}{\uparrow}$$

3.3   Benefit-Cost Ratio

The benefit-cost ratio (or B/C ratio) is defined as follows:

a) $\text{B/C ratio} = \dfrac{\text{Present worth of total benefits}}{\text{Present worth of total costs}}$ or

b) $\text{B/C ratio} = \dfrac{\text{Equivalent annual total benefits}}{\text{Equivalent annual total costs}}$

Both definitions (a) and (b) give the same answers. Take Example 3.4 (the service reservoir project) as an example:

$$\text{B/C ratio} = \frac{107,606,800}{98,431,000} = 1.093 \qquad\qquad (\text{by definition (a)})$$

$$\text{B/C ratio} = \frac{7,000,000}{6,403,000} = 1.093 \qquad\qquad (\text{by definition (b)})$$

If the B/C ratio is greater than 1, the project is worthwhile, and vice versa. When B/C ratio is exactly 1, the project is a break-even one. A project having a positive net present value or a positive net annual benefit must have a B/C ratio greater than 1. Similarly, a project having a negative net present value or a negative net annual benefit will have a B/C ratio less than 1.

3.4   Salvage Value and Equivalent Annual Cost

If the price of a new vehicle is $200,000, and if its price drops to $40,000 six years later, the vehicle is said to have a salvage value of $40,000 after six years. In general, the salvage value of an asset is its value after n years. (Readers are reminded that the inflation-free assumption is still valid). We usually use P to denote an asset's initial cost, and R its salvage value. Diagrammatically, P and R can be presented as shown in Fig. 3.8.
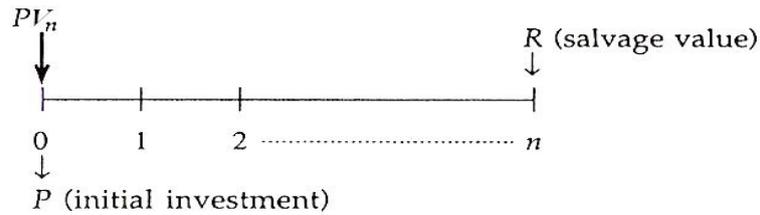
Fig. 3.8 – Present value of salvage value *R*

Let $PV_n =$ = the present value of the salvage value $R = R \times \left[\dfrac{1}{(1+i)^n} \right]$

$\therefore$ Net present value of the asset investment $= P - PV_n$

$$= P - \left[\frac{R}{(1+i)^n} \right]$$

Let the equivalent annual cost of owning an asset with salvage value *R* be *A*. *A* can be calculated by spreading the net present value into *n* yearly payments

$$\therefore A = \left[P - \frac{R}{(1+i)^n} \right] \times \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right] \qquad \text{(Eq 3.8)}$$

We can transform Equation 3.8 algebraically to:

$$A = (P - R) \times \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right] + R \times i \qquad \text{(Eq 3.9)}$$

The algebraic transformation is left to the readers. We may use either Equation 3.8 or Equation 3.9 to find *A*, but Equation 3.9 is more commonly used.

Example 3.5

A construction manager wants to purchase a new crane for his company. He is offered by two agents of two different crane manufacturers. After detailed investigation he has obtained the following data:

	Crane Model A	Crane Model B
Cost of crane	$800,000	$650,000
Annual maintenance cost	$55,000 p.a.	$70,000 p.a
Salvage value	$100,000	$65,000
Life	10 years	10 years

Assuming *i* = 8% p.a., determine for the manager which crane model he should purchase.

44

a) **Present Value Method**

	Crane Model A	*Crane Model B*
(i) Capital	**$800,000**	**$650,000**

(ii) Present value of annual maintenance

$$50{,}000 \ \times \ \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \qquad\qquad 70{,}000 \ \times \ \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$= 55{,}000 \ \times \ (6.7101) \qquad\qquad = 70{,}000 \ \times \ (6.7101)$$

$$= \mathbf{\$369{,}055} \qquad\qquad = \mathbf{\$469{,}707}$$

(iii) *Less:* present value of salvage

$$-100{,}000 \ \times \ \left[\frac{1}{(1+i)^n}\right] \qquad\qquad -65{,}000 \ \times \ \left[\frac{1}{(1+i)^n}\right]$$

$$= -100{,}000 \ \times \ (0.4632) \qquad\qquad = -65{,}000 \ \times \ (0{,}4632)$$

$$= \mathbf{-\$46{,}320} \qquad\qquad = \mathbf{-\$30{,}108}$$

(iv) Present value of total costs — **$1,122.735** — **$1,089599**

($i = 0.08$ and $n = 10$ are taken in the above calculations)

Model B has a lower present value of total costs so it should be purchased.

Download free eBooks at bookboon.com

b) **Equivalent Annual Cost Method**

	Crane Model A	*Crane Model B*
(i) Equivalent annual cost of capital	$(800{,}000 - 100{,}000) \times \left[\dfrac{i(1+i)^n}{(1+i)^n - 1} \right]$	$(650{,}000 - 65{,}000) \times \left[\dfrac{i(1+i)^n}{(1+i)^n - 1} \right]$
	$+\ 100{,}000\ \times\ i$	$+\ 65{,}000\ \times\ i$
	$=\ 700{,}000\ \times\ (0.1490) + 8{,}000$	$=\ 585{,}000\ \times\ (0.1490) + 5{,}200$
	= \$112,300	**= \$92,365**
(ii) Annual maintenance cost	**\$55,000**	**\$70,000**
(iii) Total Equivalent Annual cost	**\$167,300**	**\$162,365**

(i = 0.08 and n = 10 are taken in the above calculations)

The same conclusion can be drawn as Model B has a lower total equivalent annual cost.

3.5   Redeeming a Loan

The basic idea of compound interest is that, as seen in the previous sections, the interest is calculated *based on the new balance* for each period. If \$100,000, say, is deposited in a bank and the interest rate is 5% per annum, it becomes \$105,000 (i.e. \$100,000 × 1.05) at the end of the first year. The interest gained in the following year will be calculated based on the *new balance* \$105,000, and this sum will grow to \$110,250 (i.e. \$105,000 × 1.05) at the end of the second year, and so on. The same basic idea applies to the interest charged to a loan, that is, interest will be charged based on the *new balance* (or the unpaid balance) of the loan. Let us take a look at several different repayment methods in recovering a loan of \$100,000 in 5 years at i = 5% p.a.

NOTE: i is sometimes termed the *cost of capital* because one has to pay that percentage of money in order to obtain the capital. In this case, one has to pay \$5,000 (5%) in order to obtain a loan of \$100,000 in the first year. Another name for i is *discount rate.*

Example 3.6

Repayment Method 1: the principal sum borrowed (i.e. $100,000) be amortized in five years, $20,000 each year, as shown in Table 3.1:

	(1)	(2)	(3)	(4)	(5)
End of year	Principal borrowed	Principal amortization	Unpaid balance (i.e. new balance)	Interest charged to new balance (Col 3 × 5%)	Annual installment (Col 2 + Col 4)
0	100,000		100,000		
1		20,000	80,000	5,000	25,000
2		20,000	60,000	4,000	24,000
3		20,000	40,000	3,000	23,000
4		20,000	20,000	2,000	22,000
5		20,000	0	1,000	21,000
					∑ = 115,000

Table 3.1 – Uniform principal amortization for a loan of $100,000

Repayment Method 2: the principal sum borrowed be amortized in five years in an increasing manner from $10,000 to $30,000 as per Table 3.2.

	(1)	(2)	(3)	(4)	(5)
End of year	Principal borrowed	Principal amortization	Unpaid balance (i.e. new balance)	Interest charged to new balance (Col 3 × 5%)	Annual installment (Col 2 + Col 4)
0	100,000		100,000		
1		10,000	90,000	5,000	15,000
2		15,000	75,000	4,500	19,500
3		20,000	55,000	3,750	23,750
4		25,000	30,000	2,750	27,750
5		30,000	0	1,500	31,500
					∑ = 117,500

Table 3.2 – Non-uniform principal amortization for loan $100,000

We should note that the present values of the annual installments for Tables 3.1 and 3.2 are the same. They both equal to $100,000 taking i = 5% per annum and can be shown mathematically as follows:

Present Value (Repayment Method 1)

$$= \frac{25{,}000}{1+0.05} + \frac{24{,}000}{(1+0.05)^2} + \frac{23{,}000}{(1+0.05)^3} + \frac{22{,}000}{(1+0.05)^4} + \frac{21{,}000}{(1+0.05)^5} \quad \text{------ (Eq 3.10)}$$

= $100,000

Present Value (Repayment Method 2)

$$= \frac{15{,}000}{1+0.05} + \frac{19{,}500}{(1+0.05)^2} + \frac{23{,}750}{(1+0.05)^3} + \frac{27{,}750}{(1+0.05)^4} + \frac{31{,}500}{(1+0.05)^5} \quad \text{------ (Eq 3.11)}$$

= $100,000

Although the total amount of annual installment for Repayment Method 1 ($115,000) is less than that for Repayment Method 2 ($117,500), it would be wrong to say that Method 1 is a better alternative merely because of its lower value of total amount of annual installments. It is wrong to say so because the present values of the two methods are the same.

Now let us look at another method – Repayment Method 3.

Repayment Method 3: the principal sum borrowed to be paid in a lump sum at the end of Year 5 is as shown in Table 3.3.

End of year	(1) Principal borrowed	(2) Principal amortization	(3) Unpaid balance (i.e. new balance)	(4) Interest charged to new balance (Col 3 × 5%)	(5) Annual installment (Col 2 + Col 4)
0	100,000		100,000		
1		0	100,000	5,000	5,000
2		0	100,000	5,000	5,000
3		0	100,000	5,000	5,000
4		0	100,000	5,000	5,000
5		100,000	0	5,000	105,000
					$\sum = 125,000$

Table 3.3 – Single principal amortization for a loan of $100,000

The present value of the annual installments for this method will of course be the same again, i.e. $100,000, if i is taken as 5 percent per annum. The verification is left to the readers.

We should pay attention to columns (3) and (4) of Table 3.3. Apparently, the annual interest seems like *simple interest calculation*, giving $5,000 of interest constantly from Year 1 to Year 5. The calculation has been based on the multiplication of the principal sum by the interest rate (i.e. $P \times i = \$100,000 \times 5\%$ per annum), which looks similar to the method of simple interest calculation. However, the interest payment shown in Table 3.3 is in fact calculated by compound interest principle, with the *new balances* (i.e. column 3) remain unchanged at $100,000, the same as the principal, over the whole period of loan recovery of five years. In such a case, the interest is actually charged annually to the *new balance* (i.e. compound interest principle), as if it is charged to the principal sum.

3.6   Principal Amortization and Interest Payment of a Loan

In redeeming a loan, the redeeming amount is divided into two parts: (1) principal amortization, and (2) interest payment. The first part principal amortization does not include interest at all. The total amount amortized during the total period of the loan should be exactly equal to the loan itself. The words "amortized" and "amortization" have already appeared in Example 3.6. The second part interest payment is for paying the "cost of capital" (see the *NOTE* immediately before Example 3.6). We are going to illustrate these in Example 3.7.

Example 3.7

A company has borrowed $100,000 from a bank which charges the former 5% interest p.a. The loan has to be recovered in 5 years compounded annually.

a) How much should the company pay at the end of each year to the bank (assuming uniform payment)?

b) The bank changes the interest rate to 4% p.a. at the beginning of the third year. What will be the amount of the company's last payment (i.e. payment at the end of year 5) if it keeps on paying the bank the same amount as calculated in (a) above at the end of years 3 and 4?

c) Continued from (b), what will be the company's repayment schedule if it chooses to pay back the bank in the form of uniform payments at the end of years 3, 4 and 5?

Answer:

a) Yearly payment $= 1,000,000 \times \left[\dfrac{i(1+i)^n}{(1+i)^n - 1} \right]$ (for $i = 0.05$ and $n = 5$)

$= 100,000 \times 0.2309748$

$= \$23,097.48$

End of year	Principal unpaid	Yearly payment	Interest	Principal amortization	Remaining principal
1	$100,000.00	$23,097.48	$5,000.00	$18,097.48	$81,902.52
2	$81,902.52	$23,097.48	$4,095.12	$19,002.36	$62,900.16
3	$62,900.16	$23,097.48	$3,145.01	$19,952.47	$42,947.69
4	$42,947.69	$23,097.48	$2,147.38	$20,950.10	$21,997.59
5	$21,997.59	$23,097.48	$1,099.89	$21,997.59	$0.00

In the above table, for year 1, interest $= 100,000 \times 5\% = 5,000$. The principal amortization $= 23,097.48 – 5,000 = 18,097.48$, and therefore the remaining principal $= 100,000 – 18,097.48 = 81,902.52$. For year 2, interest $= 81,902.52 \times 5\% = 4,095.12$, and the principal amortization $= 23,097.48 – 4,095.12 = 19,002.36$, and hence the remaining principal $= 81,902.52 – 19,002.36 = 62,900.16$. Similar calculations are applied to years 3, 4 and 5. The final remaining principal would naturally be equal to zero at the end of year 5.

It is interesting to note that the five figures under the column **Principal amortization** sum up to exactly the loan amount $100,000. The same is true for parts (b) and (c) below.

b) In the part (b) table, only the first two rows (years 1 and 2) are similar to the part (a) table (5% p.a. interest rate). For years 3, 4 and 5, interest = principal unpaid × 4%.

End of year	Principal unpaid	Yearly payment	Interest	Principal amortization	Remaining principal
1	$100,000.00	$23,097.48	$5,000.00	$18,097.48	$81,902.52
2	$81,902.52	$23,097.48	$4,095.12	$19,002.36	$62,900.16
3	$62,900.16	$23,097.48	$2,516.01	$20,581.47	$42,318.69
4	$42,318.69	$23,097.48	$1,692.75	$21,404.73	$20,913.96
5	$20,913.96	$21,750.52	$836.56	$20,913.96	$0.00

So, if the company keeps on paying the bank the same amount as calculated in (a) at the end of years 3 & 4, the amount of the company's last payment is $21,750.52 ($836.56 interest already included).

c) Because the interest rate changes to 4% p.a. at the beginning of the 3rd yr, the new uniform payment is:

$$\$62,900.16 \times \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right] \text{ (for } i = 0.04 \text{ and } n = 3)$$

$$= \$62,900.16 \times 0.3603485$$

$$= \$22,665.98$$

If the company chooses to pay back the bank in the form of uniform payments at the end of years 3, 4 & 5, the repayment schedule will be like this:

End of year	Principal unpaid	Yearly payment	Interest	Principal amortization	Remaining principal
1	$100,000.00	$23,097.48	$5,000.00	$18,097.48	$81,902.52
2	$81,902.52	$23,097.48	$4,095.12	$19,002.36	$62,900.16
3	$62,900.16	$22,665.98	$2,516.01	$20,149.97	$42,750.19
4	$42,750.19	$22,665.98	$1,710.01	$20,955.97	$21,794.22
5	$21,794.22	$22,665.98	$871.76	$21,794.22	$0.00

Exercise Questions for Chapter 3

Exercise Question 1

The pavement of a road requires $400,000 per year to maintain. The feasibility of a new pavement is being considered for reducing maintenance costs. If the new pavement needs no maintenance in the first three years, then $200,000 per year for the next seven years, and then $400,000 per year thereafter, what is the immediate expenditure for the new pavement that is justifiable? (Assume a discount rate of 10% p.a.).

Exercise Question 2

A contractor borrowed $500,000 from a bank to buy earth-moving equipment with an estimated service life of 10 years. The bank charged the contractor 12% interest p.a. and required him to pay back the loan in 10 years' time.

a) Assuming that the contractor paid back the bank in 10 equal instalments (once every year), calculate the amount of each end-of-year payment.

b) The contractor at the end of year 4 wished to make an early redemption (i.e. pay all the money that he owed the bank). How much should he pay?

c) The bank negotiated with the contractor and reduced the interest rate to 10% p.a. at the beginning of the 5th year in order to attract the contractor to stay borrowing. What would be the contractor's repayment schedule if he chose to pay back the bank in the form of six uniform payments from the end of years 5 to the end of year 10?

d) If the bank changed the interest rate back to 12% p.a. at the beginning of the 8th year, what would be the amount of the contractor's last payment (i.e. payment at the end of year 10) if he kept on paying the bank the same instalment as calculated in (c) above at the end or years 8 and 9?

Exercise Question 3

There are two alternatives to construct a storage house. Both serve the purpose of allowing construction materials to be stored in the house. However, due to different construction methods (one is made of wood and the other made of bricks), different life spans and cash flow patterns are associated with each alternative as follows:

	Alternative 1 (wood)	Alternative 2 (bricks)
Life	10 years	15 years
Initial capital cost	$900,000	$1,300,000
Operation and maintenance cost	$80,000 p.a.	$20,000 p.a.

Assuming the discount rate to be 16% p.a., choose the better alternative by:

a) the present value method, and

b) the equivalent annual cost method.

(Hints: compare the alternatives based on the same number of years, i.e. 30 years)

Exercise Question 4

We have seen from Section 3.5 (or Example 3.6) of Chapter 3 that no matter what methods we use to pay back a loan, the sum of the present values of the payments is a constant value, equal to the principal amount borrowed. Prove mathematically to show that such a phenomenon is always true.

Exercise Question 5

We have seen from Section 3.6 (or Example 3.7) of Chapter 3 that the number is zero at the bottom-right corner of the table in Part a) of the example. Prove mathematically to show that this is always true for all cases in general.

4 Internal Rate of Return (IRR) and the Differences between IRR and NPV

4.1 Net Cash Flow (NCF)

Usually, we present the cash flows of a capital project investment in a Cash Flow Table in the following manner:

End of year	Cash-out	Cash-in	NCF
0	C_0	B_0	$NCF_0 = B_0 - C_0$
1	C_1	B_1	$NCF_1 = B_1 - C_1$
2	C_2	B_2	$NCF_2 = B_2 - C_2$
:	:	:	:
:	:	:	:
n	C_n	B_n	$NCF_n = B_n - C_n$

Table 4.1 – Cash flow table and net cash flow (NCF)

Net Cash Flow (NCF in short) of a certain year is defined as (Cash-in – Cash-out) in that year. The NPV (net present value), which has been discussed in Section 3.2 of Chapter 3, can be calculated by Equation 4.1 as follows:

$$\text{NPV} = \frac{\text{NCF}_0}{1} + \frac{\text{NCF}_1}{1+i} + \frac{\text{NCF}_2}{(1+i)^2} + ... + \frac{\text{NCF}_n}{(1+i)^n} \qquad \text{(Eq 4.1)}$$

As shown in Example 3.4 of Chapter 3, we can use NPV to find whether or not a project is viable. If the value of NPV is positive, then the project is viable, and vice versa. When such a method is used, we need to have a pre-determined i value to calculate the NPV.

In this chapter, however, we will use a different approach to find whether or not a project is viable. The Discount Cash Flow (DCF in short) method will be introduced, and this method associates with the Internal Rate of Return (IRR in short). We will see these in Section 4.2.

4.2   DCF Method and IRR

In the DCF (Discount Cash Flow) method, we do not need to have a pre-determined i. Instead of using Equation 4.1 to find the NPV, we set NPV equal to zero, that is:

$$\frac{\text{NCF}_0}{1} + \frac{\text{NCF}_1}{1+i} + \frac{\text{NCF}_2}{(1+i)^2} + ... + \frac{\text{NCF}_n}{(1+i)^n} = 0 \qquad \text{(Eq 4.2)}$$

We then solve Equation 4.2 and find the value of i. The solution of the variable i is called the IRR – internal rate of return. Equation 4.2, however, is difficult to solve, especially when n is large. Therefore, we rarely solve the equation algebraically, but usually use a numerical method, called DCF method, to find the solution of i in Equation 4.2. The following example illustrates how we can use DCF method to find IRR.

Example 4.1

Find the IRR of an investment of $50,000 whose receipts in the next four years are $15,000, $15,000, $20,000 and $20,000 respectively. Is this investment viable if the minimum desirable rate of return is 10% p.a.?

To calculate the IRR, we do it in a table as shown in Table 4.2:

End of Year	(1) Cash out	(2) Cash in	(3) NCF	(4) $\left[\dfrac{1}{(1+i)^n}\right]$ $i=12\%, n= 0$ to 4	(5) $(DCF)^{12\%}$ $(3)\times(4)$	(6) $\left[\dfrac{1}{(1+i)^n}\right]$ $i=16\%, n= 0$ to 4	(7) $(DCF)^{16\%}$ $(3)\times(6)$
0	50,000		-50,000	1.0000	-50,000	1.0000	-50,000
1		15,000	15,000	0.8929	13,393	0.8621	12,931
2		15,000	15,000	0.7972	11,958	0.7432	11,148
3		20,000	20,000	0.7118	14,236	0.6407	12,814
4		20,000	20,000	0.6355	12,710	0.5523	11,046
				NPV =Σ= 2,297		NPV =Σ= -2,061	

Table 4.2 – IRR calculation by DCF method

The first four columns in Table 4.2 (including 'End of Year', (1), (2) and (3)) are the same as Table 4.1. The 12% p.a. in Column (4) is entered arbitrarily, and the figures in this column can be calculated using the present value formula or simply be copied from the Appendix. In Column (5), $(DCF)^{12\%}$ tells us the PV (present value) of each row of NCF at a 12% p.a. discount rate. Column (6) is similar to Column (4) and Column (7) is similar to Column (5), except that 16% p.a. instead of 12% p.a. discount rate is used this time. This 16% p.a. is also entered arbitrarily. The sum of the figures in Column (5) is 2,297, and this represents the NPV (net present value) of the NCFs at $i = 12\%$ p.a. (see Equation 4.1). Similarly, Column (7) tells us that the NPV of the NCFs is -2,061 at $i = 16\%$ p.a.

So, if $i = 12\%$ p.a., NPV = 2,297, and if $i = 16\%$ p.a., NPV = -2,061. There exists a value of i where 12% p.a. $< i <$ 16% p.a. such that the NPV would be equal to zero, satisfying Equation 4.2. So, this value of i, by its mathematical definition, is the IRR, whose approximate value can be calculated by the "similar triangles method" as follows:

$$i = 12\% + \left(\frac{2,297}{2,297 + 2,061}\right) \times (16 - 12)\% = 14.11\% \text{ p.a.}$$

The above IRR (14.11% p.a.) is not very accurate because by similar triangles method we have assumed the NPV vs. i curve to be linear between the interval 12% and 16% of the i axis, but in fact it is not (Equation 4.1 is not linear). We can get a more accurate answer by narrowing down the interval, say, between 13.5% and 14.5% (instead of 12% and 16%) and carry out the whole process of Table 4.2 again. A more accurate answer will be obtained by doing this second iteration. Computer programs have been developed to be able to calculate a very accurate IRR by performing much such iteration. This was why we entered 12% and 16% (or any other percentages) arbitrarily at the first instant; we can finally obtain the same IRR anyway.

In Microsoft Excel, a built-in function is provided for calculating IRR. For Example 4.1, we can enter -50000 in Cell A1 (or any other cell we like). If we have entered it in Cell A1, then we have to enter 15000, 15000, 20000 and 20000 in Cells A2, A3, A4 and A5 respectively. If we want to have the answer (a very accurate IRR) in Cell A7 (or any other preferred cell), then in this cell we enter =IRR(A1:A5) and then press Enter. A very accurate IRR answer will appear in this cell, and in this example it is 14.04%. Another built-in function of Microsoft Excel allows us to calculate NPV. If we enter =NPV(12%,A2:A5)+A1 in a cell and then press Enter, then the NPV 2,297 (see Table 4.2) will appear in this cell.

So far we have discussed the mathematical definition of IRR and the method to obtain it. But what does IRR mean in practice, that is, what does 14.04% p.a. represent? The following explains it. If we borrow $50,000 from a bank, and pay the bank $15,000 after one year, $15,000 after two years, $20,000 after three years, and $20,000 after four years, then the bank is actually charging us 14.04% p.a. interest rate. Or we may put it in another way: if we put $50,000 in a business, and the net incomes in the next four years are $15,000, $15,000, $20,000 and $20,000 respectively, then the rate of return of our investment is 14.04% p.a. This 14.04% p.a. is a constant rate throughout the whole period of four years. This is the practical meaning of IRR. Since the IRR is 14.04% p.a. and is greater than the minimum desirable rate of return 10% p.a. (given by the question), the investment is viable.

As a reminder, NPV method can also be used to find the viability of this investment. We can use Equation 4.1 with i = 10% p.a. (the minimum desirable rate of return) to calculate the NPV. If the NPV is positive, the investment is viable, and vice versa. In this example, NPV is of course positive.

One point we should take note is that DCF method takes account of depreciation automatically inasmuch as it allows for the capital investment to be offset against the incomes over the life of the investment. What follows is a very simple example illustrating this point. If we invest $1,000 and the life of the investment is 1 year, and the income is $1,100 at the end of year 1 (i.e. the end of the life of the investment), then the gain is $100 and the IRR is 10% p.a. So we can see that $1,000 has depreciated in this 1 year. In performing financial analysis, we have to be careful that depreciation is not double-counted, since the DCF method takes account of depreciation automatically. We will see such examples in Chapter 7.

4.3 IRR versus NPV

The NPV (net present value) and the IRR (internal rate of return) are the two most common and important indicators for capital investments. These two indicators, however, have intrinsic differences between one and the other. Some people take the view that IRR is more cognitively efficient than NPV, because IRR is expressed as a percentage (or a rate of return) while NPV is just a monetary value cognitively inefficient to decision makers. However, some people take the view that NPV is more conceptually "correct" despite the fact that IRR may be more popular than NPV, and that NPV is more theoretically sound because IRR may be too "capricious" or "fickle" and sometimes does not rank some projects in the same order as NPV does. We have to examine this in detail.

If there are two alternatives (A and B, say) for us to rank (or to prioritize), we may use either (1) highest IRR criterion, or (2) highest NPV criterion. It is important to note that the results of the two criteria may be consistent with each other or may be not. The NPV vs. i curves of Alternative A and Alternative B are plotted in Fig. 4.1. We should note that the two curves representing the two alternatives are not linear but for convenience sake we just draw them linear.
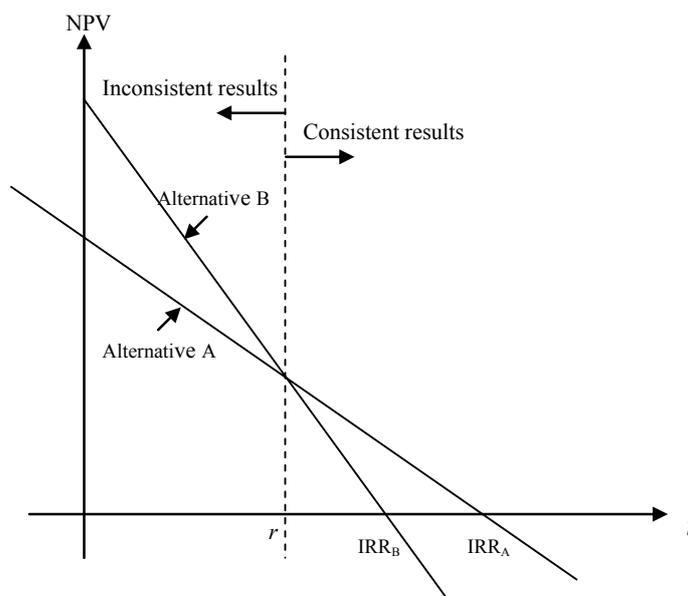
Figure 4.1 – Potential inconsistency of NPV and IRR

From Fig. 4.1, Alternative A is better than Alternative B if criterion (1), highest IRR, is used, since IRR_A is greater than IRR_B. For $i > r$, the NPV of Alternative A is higher than that of Alternative B so according to criterion (2), highest NPV, Alternative A is better than Alternative B. Hence, the two criteria give consistent ranking. For $i < r$, however, the NPV of Alternative B is higher than that of Alternative A, but IRR_A is greater than IRR_B, hence the two criteria give inconsistent ranking. Let us see Example 4.2.

Example 4.2

This is an illustrative example of Fig 4.1. *n* is taken as 30 years for this example.

	Alternative A	Alternative B
Initial capital	$5,000,000	$7,000,000
Annual operation/maintenance cost	$40,000	$100,000
Annual benefit	$560,000	$800,000

_____

NPV (assume i = 6% p.a.)   = $2,157,000 $2,635,000

      (Alternative B has a higher NPV than Alternative A)

IRR   =   9.8% p.a.   9.3% p.a.

      (Alternative A has a higher IRR than Alternative B)

The highest NPV criterion is inconsistent with the highest IRR criterion

_____

NPV (assume i = 9% p.a.)   = $342,000 $192,000

      (Alternative A has a higher NPV than Alternative B)

IRR   =   9.8% p.a.   9.3% p.a.

      (Alternative A has a higher IRR than Alternative B)

The highest NPV criterion is consistent with the highest IRR criterion

_____

The value of *r* (defined in Fig. 4.1) must be between 6% p.a. and 9% p.a., and is approximately 8.5% p.a. in this example.

The use of NAB, as discussed in the last part of Section 3.2 of Chapter 3, would have a ranking similar to that of NPV. The use of B/C ratio, as defined in Section 3.3 of Chapter 3, would have a ranking similar to that of IRR, because B/C is a ratio and somehow reflects a kind of 'rate of return', though it is not the same as IRR. The verification of the above statements is left as exercise to the readers.

In comparing multiple alternatives, we should use the highest NPV criterion and not the highest IRR criterion. The highest IRR criterion may lead to wrong conclusion. The next example (Example 4.3) and the next section (Section 4.4) discuss why it is so in detail.

Example 4.3

This example is an extension of Example 4.2. In addition to Alternatives A and B, we now have Alternative C. Their details are shown below:

	Alternative A	Alternative B	Alternative C
Initial capital	$5,000,000	$7,000,000	$11,000,000
Annual operation/maintenance cost	$40,000	$100,000	$150,000
Annual benefit	$560,000	$800,000	$1,050,000

n is taken as 30 years for this example, and the minimum desirable rate of return is 6% p.a.

The NPV ranking and the IRR ranking of the three alternatives are shown in Table 4.3.

	Alternative A	Alternative B	Alternative C
NPV (at 6% discount rate)	$2,157,000	$2,635,000	$1,388,000
Ranking of NPV	2nd	1st	3rd
IRR	9.8% p.a.	9.3% p.a.	7.2% p.a.
Ranking of IRR	1st	2nd	3rd

Table 4.3 – NPV and IRR Rankings of Alternatives A, B and C

We can see that NPV and IRR methods give us different rankings of the alternatives. As said above, the NPV ranking is the correct ranking. The IRR ranking leads to wrong conclusion.

However, there is a method called Incremental IRR Analysis that will give us the result consistent with the NPV ranking (correct ranking). We are going to see how the incremental IRR analysis is used to rank the three alternatives.

First, we arrange the alternatives in an increasing order of initial capitals. In Example 4.3, we have already done this because the initial capitals of Alternatives A, B and C are $5,000,000, $7,000,000 and $11,000,000 respectively, already in an increasing order. (Otherwise, we would have to change A, B and C to, say, B, C and A, or whatever they might be).

Second, we compare Alternative A and Alternative B. The better alternative is determined by comparing the incremental IRR with the predetermined discount rate (or the minimum desirable rate of return) which is 6% p.a. in this example. The definition of incremental IRR is simply the IRR calculated from the incremental NCF. The incremental NCF is the difference of the NCF of Alternatives A and that of Alternative B, or more exactly, NCF of Alternative B minus NCF of Alternative A. If the incremental IRR is larger than 6% p.a., then Alternative B is better than Alternative A, and vice versa. Let us see how we do this below.

End of year	NCF_B		End of year	NCF_A		End of year	Incremental NCF_{B-A}
0	-7,000,000		0	-5,000,000		0	-2,000,000
1	700,000		1	520,000		1	180,000
2	700,000		2	520,000		2	180,000
.	.	**minus**	.	=	.		.
.	.		.		.		.
.	.		.		.		.
30	700,000		30	520,000		30	180,000

As defined before, the incremental IRR is the IRR of the Incremental NCF_{B-A}, and is found to be 8.1% p.a. Since 8.1% p.a. is larger than 6% p.a., the minimum desirable rate of return, Alternative B is better than Alternative A. So, Alternative A is out and Alternative B is left behind for further consideration.

Third, we compare Alternative B and Alternative C. We repeat the process in a similar way to see whether Alternative C is better than Alternative B, or vice versa. This is done below.

End of year	NCF_C		End of year	NCF_B		End of year	Incremental NCF_{C-B}
0	-11,000,000		0	-7,000,000		0	-4,000,000
1	900,000		1	700,000		1	200,000
2	900,000		2	700,000		2	200,000
.	.	**minus**	.	=	.	.	.
.	.		.		.	.	.
.	.		.		.	.	.
30	700,000		30	700,000		30	200,000

The incremental IRR of the incremental NCF_{C-B} is found to be 2.8% p.a. Since 2.8% p.a. is smaller than 6% p.a., Alternative B is better than Alternative C. So, Alternative B is the best overall. Therefore, the incremental IRR ranking is consistent with the NPV ranking. In the following section, we will see why NPV is more reliable in engineering economy sense.

4.4   IRR as Financial Indicator and NPV as Economic Indicator

As said before, IRR and NPV have intrinsic differences between one and the other. There is a more fundamental definition given to the two. The definition is: NPV is an economic indicator and IRR a financial indicator. In other words, NPV gives the society's point of view and IRR the private investor's point of view. Because IRR functions as a financial indicator, its value varies with the change of financial arrangement (e.g. change of equity-loan ratio) of a capital investment. NPV, however, does not vary when financial arrangement varies, because it functions as an economic indicator. Example 4.4 illustrates this.

Example 4.4 – an illustrative example

If an investor has an all-equity case investment (i.e. no borrowing from bank; the capital is totally provided by the investor) in which an initial capital outlay of $10,000 leads to a receipt of $5,000 each year for three years as shown in Table 4.4, the investor obtains an IRR of 23.4% p.a.

End of Year	Cash-out	Cash-in
0	10,000	
1		5,000
2		5,000
3		5,000

Table 4.4 – Cash Flows of An Investment

If the investor uses $4,000 as the equity of the investment and borrows $6,000 as a loan (paying 10% interest per annum), making a total of $10,000 serving as the initial capital outlay of **the same investment**, then the net annual receipts in the next three years, having deducted the three annual principal amortizations and the annual interest payments from the gross annual receipts, are calculated to be $2,400, $2,600 and $2,800 as shown in Table 4.5 below.

End of Year	(1) Amount borrowed	(2) Principal amortization	(3) Balance of Principal unpaid	(4) Interest (10%)	(5) Total annual Payment (2) + (4)	(6) Net annual receipt 5,000 - (5)
0	6,000		6,000			
1		2,000	4,000	600	2,600	5,000-2,600=2,400
2		2,000	2,000	400	2,400	5,000-2,400=2,600
3		2,000	0	200	2,200	5,000-2,200=2,800

Table 4.5 – Net Annual Receipts for Uniform Principal Amortization of $2,000 for Loan $6,000

In other words, the investor will obtain $2,400, $2,600 and $2,800 in Years 1, 2 and 3 respectively if he initially invests $4,000 (the equity). The cash flows of the investment become those as shown in Table 4.6. It should be noted Table 4.4 and Table 4.6 represent **the same investment**, although they have different financial arrangements. One should have no advantage over another as far as the **economic value of the investment** is concerned.

End of Year	Cash-out	Cash-in
0	4,000	
1		2,400
2		2,600
3		2,800

Table 4.6 – Cash Flows with $4,000 as Equity and $6,000 as Loan

The IRR is 41.2% p.a. for the cash flows of Table 4.6, which is higher than 23.4% p.a. as calculated from the cash flows shown in Table 4.4. This indicates that the financial arrangement of $4,000 equity and $6,000 loan is **financially better** than the all-equity case, although the **economic value of the investment is unchanged** (see below). If the NPV method is used to compare these two sets of cash flows, namely, Tables 4.4 and 4.6, or (-10,000, 5,000, 5,000, 5,000) and (-4,000, 2,400, 2,600, 2,800), using i = 10% p.a. (the borrowing interest rate), the NPVs will be the same as shown in the following calculations:

NPV of the cash flows of Table 4.4

$$= -10,000 + \frac{5,000}{(1+i)} + \frac{5,000}{(1+i)^2} + \frac{5,000}{(1+i)^3} = 2,434.26 \qquad \text{(Eq 4.3)}$$

NPV of the cash flows of Table 4.6

$$= -4,000 + \frac{2,400}{(1+i)} + \frac{2,600}{(1+i)^2} + \frac{2,800}{(1+i)^3} = 2,434.26 \qquad \text{(Eq 4.4)}$$

Now, a further case is to be investigated. If the investor uses only $1,000 as equity and borrows $9,000 as loan at 10% p.a. interest rate, the cash flow will become (-1,000, 1,100, 1,400, 1,700), and the IRR calculated is 113.1% p.a., which is much higher than 23.4% p.a. (calculated from Table 4.4) and also 41.2% p.a. (calculated from Table 4.6). The NPV, however, is the same (i.e. $2,434.26). Verification of the NPV is left to the readers.

A mathematical proof for the NPV to remain constant in different financial arrangement situations as shown in Example 4.4 can be found from the following journal article:

Tang, S.L. and Tang, H. John (2003). The variable financial indicator IRR and the constant economic indicator NPV. *Engineering Economist*. Vol. 48, No. 1, pp. 69–78.

We have seen that the IRR becomes higher if a bigger loan is borrowed, although the economic value (indicated by the NPV) of the investment remains the same in all situations. This is because the borrowing interest rate 10% p.a. is smaller than the all-equity case IRR 23.4% p.a. If the borrowing interest rate is not 10% p.a. but higher than the all-equity IRR, then an exactly opposite effect will occur, that is, the higher the loan is, the lower the IRR will be.

To summarize, the NPV of a capital investment is constant in all situations and is not affected by the variation of financial arrangements. IRR, however, varies with the change of financial arrangements. NPV is an **economic indicator** for a capital investment, as the economic value of an investment does not change even its financial arrangement changes. The IRR, however, is suitable for use as a **financial indicator**, since private investors usually like to play around with financial arrangements to optimise their rates of return.

Exercise Questions for Chapter 4

Exercise Question 1

An appraisal of three alternative investments, A, B and C is being made and the minimum desirable rate of return is 10% p.a. on its invested capital. The details of the investments are shown below.

	Investment A	Investment B	Investment C
Initial cost	$900,000	$1,600,000	$3,100,000
Salvage value	Nil	Nil	$300,000
Net annual income	$368,000	$612,000	$846,000
Life	4 years	4 years	5 years

a) Find the IRR of each investment.

b) Find the NPV of each investment. Compare the ranking with (a) above.

c) Use the Incremental IRR method to determine which investment is the best to invest.

Exercise Question 2

Search the following paper:

Tang, S.L. and Tang, H. John (2003). "The variable financial indicator IRR and the constant economic indicator NPV". *Engineering Economist*, Volume 48, Number 1, pages 69–78.

Then read it and see if you have understood more on Section 4.4 of this chapter.

5   Effect of Inflation

5.1   Introduction

In Chapters 3 and 4, all discussions are based on the assumption that we are living in an inflation-free world. However, such an assumption is not true. The world is not inflation-free. In this chapter, we are going to remove this unrealistic assumption, that is, the first paragraph of Section 3.2 of Chapter 3 that i (real rate) is equal to i' (nominal rate or apparent rate) is no longer true.

5.2   Real IRR and Its Calculation in a World with Inflation

Section 4.2 of Chapter 4 shows that in an inflation-free world the real IRR can be calculated by solving Equation 4.2, which is reproduced below and renamed as Equation 5.1.

$$\frac{\text{NCF}_0}{1} + \frac{\text{NCF}_1}{1+i} + \frac{\text{NCF}_2}{(1+i)^2} + ... + \frac{\text{NCF}_n}{(1+i)^n} = 0 \qquad \text{(Eq 5.1)}$$

However, in a world with inflation, the real IRR must be calculated from Equation 5.2 as follows:

$$\frac{\text{NCF}_0}{1} + \frac{\text{NCF}_1}{(1+i)(1+f_1)} + \frac{\text{NCF}_2}{(1+i)^2(1+f_1)(1+f_2)} + \cdots + \frac{\text{NCF}_n}{(1+i)^n(1+f_1)(1+f_2) + ...(1+f_n)} = 0 \quad \text{(Eq 5.2)}$$

where f_1 = inflation expressed as a percentage in year 1

f_2 = inflation expressed as a percentage in year 2

:   :   :   :

f_n = inflation expressed as a percentage in year n

and NCF_0 = the actual net cash flow of year 0 (year the investment was made)

NCF_1 = the actual net cash flow of year 1 in a world with inflation

NCF_2 = the actual net cash flow of year 2 in a world with inflation

:   :   :   :

NCF_n = the actual net cash flow of year n in a world with inflation

The effect of i devalues the future cash flows. The effect of f (inflation), similar to i, also devalues the future cash flows. The factors $(1 + i)$ and $(1 + f)$, therefore, are both put as denominators in Equation 5.2.

If inflation is expected to be the same in every year within the life of the project, then:

$$f_1 = f_2 = \ldots = f_n = f$$

Equation 5.2 then becomes:

$$\frac{\text{NCF}_0}{1} + \frac{\text{NCF}_1}{(1+i)(1+f)} + \frac{\text{NCF}_2}{(1+i)^2(1+f)^2} + ... + \frac{\text{NCF}_n}{(1+i)^n(1+f)^n} = 0 \qquad \text{(Eq 5.3)}$$

f can also be regarded as the average inflation rate within those n years. The IRR (solution of i) calculated from Equation 5.3 is real. The apparent (or nominal) IRR can be calculated from Equation 5.4. The solution of i' from Equation 5.4 is of course apparent and not real.

$$\frac{\text{NCF}_0}{1} + \frac{\text{NCF}_1}{1+i'} + \frac{\text{NCF}_2}{(1+i')^2} + ... + \frac{\text{NCF}_n}{(1+i')^n} = 0 \qquad \text{(Eq 5.4)}$$

By comparing Equations 5.3 and 5.4, we can observe that:

$$(1 + i') = (1 + i)(1 + f) \qquad \text{(Eq 5.5)}$$

$$\text{or} \quad i' = (1 + i)(1 + f) - 1 \qquad \text{(Eq 5.5a)}$$

From Equation 5.5a, $i' = 1 + i + if + f - 1$

$$\text{i.e. } i' = i + if + f$$

If i and f are small, then $i \times f$ is nearly equal to zero, and so

$$i' = i + f \qquad \text{(Eq 5.6)}$$

Note that Equation 5.6 is Equation 3.2 in Section 3.1 of Chapter 3. It is an approximation of Equation 5.5a if i and f are small. Equation 5.5a is the true one.

Example 5.1

An asset was purchased four years ago at $60,000 and had a life of 4 years. This investment resulted in **actual** annual cash receipts of $14,000, $18,000, $23,000 and $28,000 respectively in the past 4 years. These figures are found from the accounting record of each year in the past 4 years. The average inflation rate in these four years was 3% p.a. Find the real IRR.

Answer:

First, find the apparent (nominal) IRR i' using Equation 5.4. We use the numerical method as described in Section 4.2 of Chapter 4 to find this i'.

End of Year	(1) NCF	(2) $\left[\dfrac{1}{1+i^{\,n}}\right]$ $i = 8\% \;\&\; n = 0\text{-}4$	(3) $(DCF)^{8\%}$ $(1) \times (2)$	(4) $\left[\dfrac{1}{1+i^{\,n}}\right]$ $i = 13\% \;\&\; n = 0\text{-}4$	(5) $(DCF)^{13\%}$ $(1) \times (4)$
0	-60,000	1.00000	-60,000	1.00000	-60,000
1	14,000	0.9259	12,963	0.8850	12,390
2	18,000	0.8573	15,431	0.7831	10,963
3	23,000	0.7938	18,257	0.6931	15,941
4	28,000	0.7350	20,580	0.6133	17,172
			7,231		-3,534

Table 5.1 – DCF Method for finding the IRR of Example 5.1

$$i' = 8\% + \left(\frac{7{,}231}{7{,}231 + 3{,}534} \right) \times (13 - 8)\% = 11.36\% \text{ (i.e. apparent IRR).}$$

Using Equation 5.5a,

$$i' = (1 + i)(1 + f) - 1$$

or $\quad 0.1136 = (1 + i)(1 + 0.03) - 1$

Solving for i gives $i = 8.12\%$

Therefore, the real IRR is 8.12% p.a. (answer).

5.3   When Is Inflation Adjustment Considered Or Not Considered?

Suppose we look back to a finished 10-years investment (2002 to 2012, say) and from a search of the accounting records we have the data as shown in Table 5.2.

End of year	Net cash flow
0 (2002)	NCF_0 (based on 2002 money value)
1 (2003)	NCF_1 (based on 2003 money value)
2 (2004)	NCF_2 (based on 2004 money value)
: :	: :
10 (2012)	NCF_{10} (based on 2012 money value)

Table 5.2 – NCFs of money value in respective years

The cash flows are **actual** transactions of those ten years so NCF_0 represents the money value of 2002, NCF_1 represents the money value of 2003, NCF_2 represents the money value of 2004, and so on. This is only possible when the accounting records are kept because reference to the records is essential in getting these data. Example 5.1 is of this kind of data. For such cash flow data, the NPV in real terms (NPV_{real}) is calculated by Equation 5.7 as follows:

$$NPV_{real} = \frac{NCF_0}{1} + \frac{NCF_1}{(1+i)(1+f_1)} + \frac{NCF_2}{(1+i)^2(1+f_1)(1+f_2)} + \cdots$$
$$+ \frac{NCF_n}{(1+i)^n(1+f_1)(1+f_2)+...(1+f_n)} \qquad \text{(Eq 5.7)}$$

Or if $f_1 = f_2 = \ldots = f_n = f$ then:

$$NPV_{real} = \frac{NCF_0}{1} + \frac{NCF_1}{(1+i)(1+f)} + \frac{NCF_2}{(1+i)^2(1+f)^2} + ... + \frac{NCF_n}{(1+i)^n(1+f)^n} \qquad \text{(Eq 5.8)}$$

which can also be expressed as:

$$NPV_{real} = \frac{NCF_0}{1} + \frac{NCF_1}{1+i'} + \frac{NCF_2}{(1+i')^2} + ... + \frac{NCF_n}{(1+i')^n} \qquad \text{(Eq 5.9)}$$

However, if we have a 10-years plan for a project investment starting in 2013 and the NCFs are estimated based on the year of planning the project – 2013 as shown in Table 5.3.

End of year	Net cash flow
0 (2013)	NCF_0 (based on 2013 money value)
1 (2014)	NCF_1 (based on 2013 money value)
2 (2015)	NCF_2 (based on 2013 money value)
⋮ ⋮	⋮ ⋮
10 (2023)	NCF_{10} (based on 2013 money value)

Table 5.3 – NCFs of values at constant base year 2013

We only know the money value of 2013 well because we do the planning in this year. All the cash flows in the next 10 years are estimated based on the 2013 money value. We call these as 'constant base year prices'. In such a case, the NPV in real terms (NPV_{real}) is calculated by Equation 5.10 (similar to Equation 4.1 of Chapter 4) as follows:

$$NPV_{real} = \frac{NCF_0}{1} + \frac{NCF_1}{1+i} + \frac{NCF_2}{(1+i)^2} + ... + \frac{NCF_{10}}{(1+i)^{10}} \qquad \text{(Eq 5.10)}$$

The NPV calculated is real because all NCFs are of constant base year prices. It can be proved mathematically. The following shows the proof.

The cash flows shown in Table 5.3 (constant base year prices) can be changed to actual transactions as shown in Table 5.2. This is done in Table 5.4.

End of year	NCF (Actual transactions)	
0	NCF_0	(based on 2013 money)
1	$NCF_1 \times (1 + f)$	(based on 2014 money)
2	$NCF_2 \times (1 + f)^2$	(based on 2015 money)
.		.
.		.
.		.
10	$NCF_{10} \times (1 + f)^{10}$	(based on 2023 money)

Table 5.4 – NCFs of actual transactions in respective years

Substituting these into Equation 5.8, we obtain:

$$NPV_{real} = \frac{NCF_0}{1} + \frac{NCF_1 \times (1+f)}{(1+i)(1+f)} + \frac{NCF_2 \times (1+f)^2}{(1+i)^2 (1+f)^2} + \cdots + \frac{NCF_{10} \times (1+f)^{10}}{(1+i)^{10}(1+f)^{10}}$$

$$= \frac{NCF_0}{1} + \frac{NCF_1}{1+i} + \frac{NCF_2}{(1+i)^2} + ... + \frac{NCF_{10}}{(1+i)^{10}}$$

which is the same as Equation 5.10. So, we have completed the proof.

Therefore, inflation although influences both revenues and costs, but the effect of inflation on both revenues and costs will cancel out each other. There is no need to consider inflation for cash flows which are based on constant year prices (i.e. Table 5.3). In Example 3.4 of Chapter 3, we can easily see that the annual operation and maintenance cost $2.5 million and the annual income $7 million are of constant base year prices, otherwise how can they remain unchanged for 30 years? During these 30 years (life of the project), the annual operation and maintenance cost will increase with the inflation rate. The same will happen to the annual income. Inflation may indeed influence both cost and income to differing degrees, but such impacts are of short duration. Usually, the effect of inflation on both cost and income will cancel out each other in the long run to regain a balance.

5.4   Concluding Remarks

In planning construction projects, we can only use constant base year prices to estimate our cash flow data (i.e. Table 5.3). So we do not have to consider inflation in calculating the real IRR or real NPV, whether in an inflation-free world or in a world with inflation. Inflation adjustment is not necessary for cash flows of this kind – constant base year prices. Examples 3.4 and 3.5 of Chapter 3 and Examples 4.1 and 4.2 of Chapter 4 are examples of cash flows of constant base year prices.

However, Example 3.6 of Chapter 3 involves bank payments. We should note that all bank payments are NOT constant base year prices. But in this example we have not made any inflation adjustment. Why? It is because in this example we are using Equation 5.9 of this chapter (see Equations 3.10 and 3.11 of Chapter 3); bank interest rate is always nominal (i.e. i'). The NPV calculated is therefore real too.

The only situation we have to be careful is Example 5.1. The cash flows in this example are those as shown in Table 5.2. For such a case, inflation adjustment is necessary. But this situation is rare. If we really come across cash flow data like this, we must make inflation adjustment.

Exercise Questions for Chapter 5

Exercise Question 1

In what cash flow situations should inflation adjustment be made or not be made? Examples should be given in your answer.

Exercise Question 2

The following cash flows of a project are the *actual transactions* in the respective years as indicated. The average inflation rate for those years was 3% p,a, Calculate the real IRR.

End of year	Cash out	Cash in
0 (2008)	$102,500	
1 (2009)		$28,900
2 (2010)		$31,000
3 (2011)		$34,600
4 (2012)		$38,500
5 (2013)		$43,300

Exercise Question 3

There is a proposed project of constructing a self-financed highway, so the toll paid by the vehicles should be sufficient to recover the initial capital cost and all other costs that will be incurred over the life of the project. Establish a suitable charge on each vehicle passing the highway with the use of the following numerical values which are estimated based on 2014 prices.

1	Capital cost (construction cost and professional fees	$500,000,000
2	Annual operation and maintenance cost	$15,000,000
3	Annual administration cost	$20,000,000
4	Replacement cost of autopay ticket machine	$4,000,000 every 5 years
5	Estimated annual traffic flow	10,000 vehicles per day in the first 5 years; the growth rate of traffic flow is 20% every 5 years
6	Life of project	25 years
7	Desirable rate of return (real)	8% p.a.
8	Estimated average inflation rate in the next 25 years	3% p.a. (averaged)

6   Cost-Volume-Profit Analysis

6.1   Introduction

Cost-volume-profit analysis usually involves the use of Break-even charts. Therefore, it is also called Break-even analysis. The analysis and the break-even charts are to be fully discussed in this chapter. Examples on break-even chart applications will be illustrated for various situations of analyses in construction.

6.2   Variable Costs, Fixed Costs and Break-even Point

A construction company expects that its Total Revenue (TR) is large enough to cover the following costs:

a) Direct costs (e.g. materials, labour, equipment, etc.)
b) Costs of administering the company
c) Costs of marketing/advertising

So, the company expects that:
TR > (a) + (b) + (c)
Or, TR = (a) + (b) + (c) + Profit
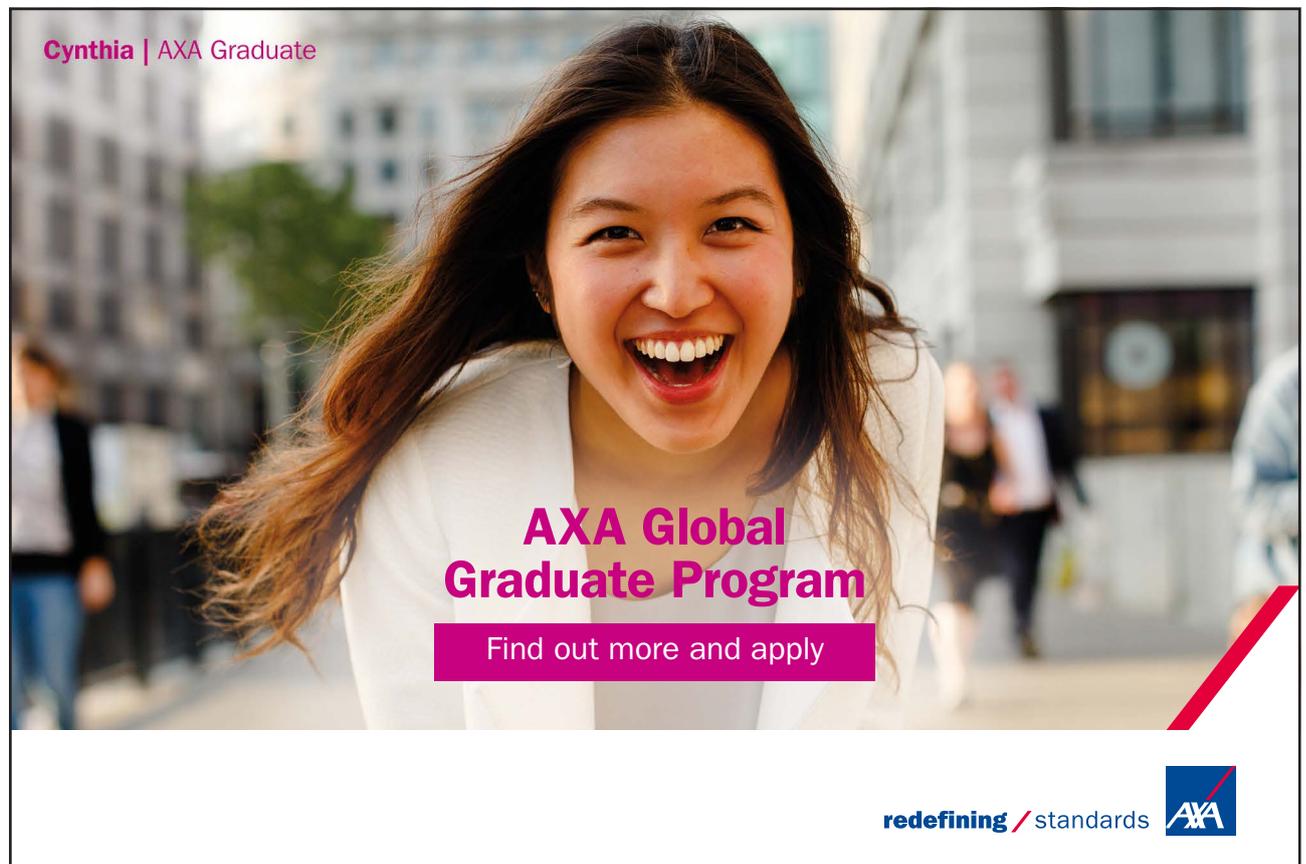
However, if TR = (a) + (b) + (c) then the company neither makes any profit nor losses any money. For such a case, the company is operating at Break-even Point (BEP in short). If Total Cost is denoted by TC, then BEP is the Volume (amount of output) of the company at which TR = TC. We will discuss 'volume' shortly, but before doing that we must explain what TC, total cost, is.

TC (total cost) is made up of VC (variable cost) and FC (fixed cost) as follows:

$$TC = VC + FC \qquad\qquad\qquad \text{(Eq 6.1)}$$

VC (variable cost) includes direct materials, direct labour, equipment, fuel, etc. Variable cost varies with volume. Let volume = x so

VC is proportional to x

Or $VC = v\,x$ (Eq 6.2)

where v = variable cost per unit of x

FC (fixed cost) is constant, and is regardless of the level of the volume. Fixed cost includes rent, interest on investment, executive salaries, insurance, property taxes, etc. In practice, we usually talk about one year's cost when we say the fixed cost amount.

Volume, as said above, is denoted by x, which is the number of units of output, usually in one year's time. When volume changes, it will affect the profit or loss of the company. The following Example 6.1 illustrates this.

Example 6.1

A construction material company makes and sells window panels. The selling price per panel is $900. The variable cost for making the window panels is $500 per unit. The fixed cost is $8,000,000. Find the BEP (break-even point).

Answer:

p = selling price per unit = $900
v = variable cost per unit = $500
FC = fixed cost = $8,000,000

We can express our analysis in Table 6.1 as follows:

Volume	$x = 18,000$	$x = 20,000$	$x = 22,000$
TR (Total Revenues)	$900 × 18,000 = $16,200,000	$900 × 20,000 = $18,000,000	$900 × 22,000 = $19,800,000
VC (Variable Cost)\n\nFC (Fixed Cost)	$500 × 18,000 = $9,000,000\n\n$8,000,000	$500 × 20,000 = $10,000,000\n\n$8,000,000	$500 × 22,000 = $11,000,000\n\n$8,000,000
TC (Total Cost)	$17,000,000	$18,000,000	$19,000,000
Net Income	($800,000)\nLoss	0\nBEP	$800,000\nProfit

Table 6.1 – Cost-volume-profit analysis (or Break-even analysis)

We can see that breaking-even occurs when the volume x is 20,000 units. If x is smaller than 20,000 units, the company will suffer a loss. If x is greater than 20,000 units, the company will have a profit. For example, if this company has a total (maximum) capacity of making 25,000 units of window panels in a year, then it will have a maximum profit of $2,000,000. Verification of it is left to the readers. Since the break-even point is at 20,000 units, we say that the BEP is at 80% of the company's capacity (i.e. 20,000 / 25,000 = 80%).
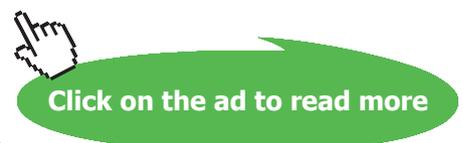
6.3   Graphical Presentation – Break-even Chart

The Break-even Chart is a graphical presentation of TR, VC, FC and TC. Fig. 6.1 below shows the break-even chart of Example 6.1.
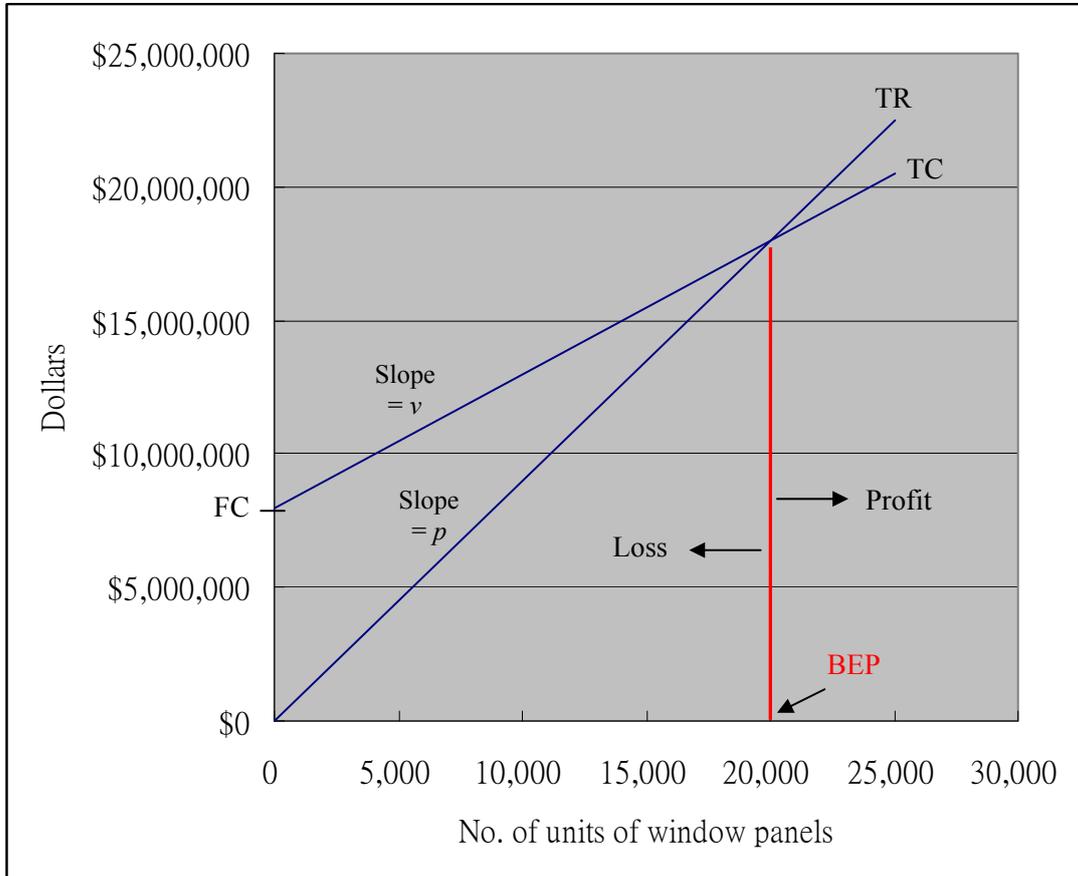
Fig. 6.1 – The break-even chart for Example 6.1

6.4   Mathematical Presentation

We are going to derive a mathematical formula for calculating the value of x at BEP.

$$\text{TR} = p\,x$$
$$\text{TC} = \text{VC} + \text{FC}$$
$$\text{Or} \quad \text{TC} = v\,x + \text{FC}$$
$$\text{At BEP,} \quad \text{TR} = \text{TC}$$
$$\text{So,} \quad p\,x = v\,x + \text{FC}$$

Hence, at BEP, $x = \dfrac{\text{FC}}{p - v}$ (Eq 6.3)

Sometimes, we express the BEP as a percentage of the capacity of a company, as already explained in Example 6.1. The following formula can be used to calculate this percentage.

$$\% \text{ capacity at BEP } = \frac{FC}{(p-v)(total-capacity-in-units)} \times 100\% \qquad \text{(Eq 6.4)}$$

Applying Equations 6.3 and 6.4 to Example 6.1, we can find the same answer as given in Section 6.2.

p = selling price per unit = \$900

v = variable cost per unit = \$500

FC = fixed cost = \$8,000,000

Using Equation 6.3, $x = \dfrac{8,000,000}{900-500} = 20,000$ units

Using Equation 6.4, % capacity of the company at BEP $= \dfrac{8,000,000}{(900-500)(25,000)}$

$$= 0.8 \text{ (or 80\%)}$$

6.5   Contribution

In Example 6.1, the construction material company sells one unit of the window panels it makes and gets \$900 for it. Since the variable cost of that unit amounts to \$500, and because \$900 – \$500 = \$400, so \$400 remains as a contribution per unit toward paying off the fixed cost. The concept of contribution is an important one in Cost-Volume-Profit analysis. Contribution is for paying off the fixed cost. If contribution can cover fixed cost, there will be profit, and vice versa. We will see an example that applies the concept of contribution.

Example 6.2

A subcontractor specialized in wastewater disposal makes and sells cast iron pipes, steel pipes and concrete pipes. The following variable costs and selling prices/sales volumes are obtained from the cost accounting department and sales department respectively.

Pipe	p	v	Volume (% of dollar sales)
Cast iron	\$3,000	\$2,000	25%
Steel	\$3,500	\$2,500	35%
Concrete	\$4,000	\$3,000	40%

The company capacity in terms of total (maximum) sales volume is \$900 million in a year. The annual fixed cost is \$200 million.

  a) Find the BEP of the subcontractor,
  b) Find the profit if the subcontractor is operating at 95% of its capacity.

Answer:

Contribution per unit expressed in percentage = $[(p - v) / p] \times 100\%$

Contribution per unit due to cast iron pipes = $[(3{,}000 - 2{,}000) / 3{,}000] \times 100\% = 33.33\%$

Contribution per unit due to steel pipes = $[(3{,}500 - 2{,}500) / 3{,}500] \times 100\% = 28.57\%$

Contribution per unit due to concrete pipes = $[(4{,}000 - 3{,}000) / 4{,}000] \times 100\% = 25.00\%$

	Contribution		% of sales		
Cast iron pipes	33.33%	×	25%	=	8.33%
Steel pipes	28.57%	×	35%	=	10.00%
Concrete pipes	25.00%	×	40%	=	<u>10.00%</u>
					28.33%

This 28.33% is the total contribution per overall sales dollars.

a) BEP = FC / Contribution = $200 m / 0.2833 = $706 m

Therefore, at BEP, the subcontractor is operating at 78.4% of capacity (i.e. 706/900=78.4%).

b) At 95% of capacity,

$$\text{Profit} = \text{TR} - \text{TC}$$
$$= (900\ m \times 95\%) - \text{VC} - \text{FC}$$
$$= 855\ m - 855\ m \times (1 - 0.2833) - 200\ m$$
$$= 855\ m - 613\ m - 200\ m$$
$$= \$42\ \text{million}$$

6.6   The Make-or-Buy Decision

We have discussed the break-even chart in Section 6.3. The concept of the break-even chart can be applied to many decision problems. From this section onward, we are going to see three different applications in three examples. The first one is a make-or-buy decision problem as shown in Example 6.3.

Example 6.3

A site agent on a new construction site has to decide whether to supply concrete for his project by setting up a batching plant on his own site or to purchase concrete from a nearby ready-mixed concrete plant. He can purchase ready-mixed concrete at $400 per cubic meter. If he wants to set up a batching plant on site, he has to spend an equivalent annual cost of $500,000 as fixed cost, and the variable cost per cubic meter of concrete he makes is $150. Should he set up a batching plant on site so as to make his own concrete?

The break-even chart is:
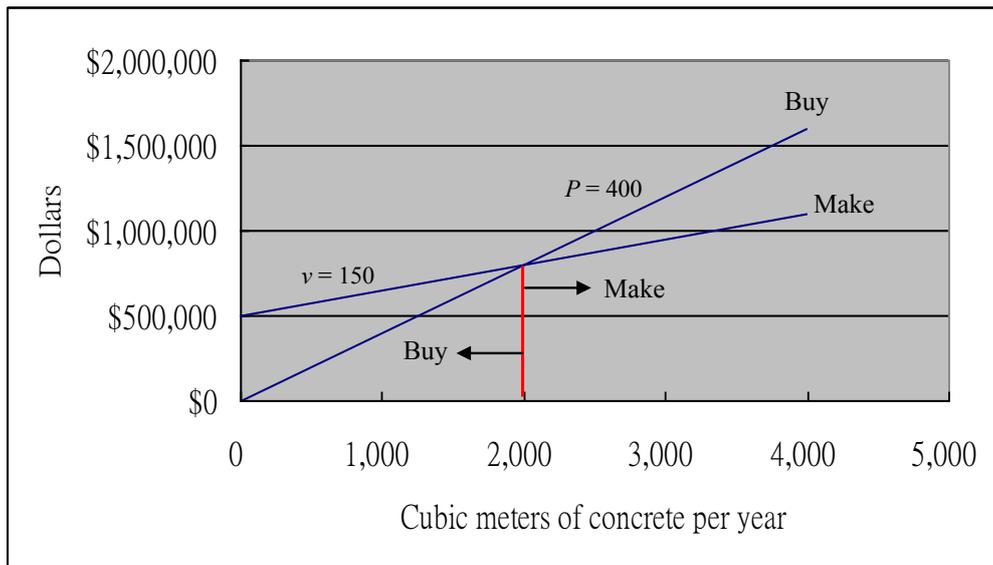
Fig. 6.2 – Break-even chart for the Make-or-Buy decision problem

The break-even chart suggests that if the site agent requires not more than 2,000 cubic meters of concrete in one year, to buy is more attractive than to make. But if he requires more than 2,000 cubic meters of concrete in a year, then to make is more attractive than to buy.

6.7   Equipment Selection Decision

The second example of applying the concept of break-even chart is on equipment selection decision. We are going to see how useful a break-even chart is.

Example 6.4

A construction company has three choices of the type of crane to be used on its sites. The choices are:

a) a fully automatic crane – this will add $900,000 a year to its fixed cost but the variable cost (hiring of operators, fuel, etc.) per hour will only be $40.

b) a semi-automatic crane – this will add $500,000 a year to its fixed cost but v (variable cost per hour) will be $200, since more operators will be needed.

c) a mainly hand operated crane – this will add $200,000 a year to its fixed cost , but v will be $500 per hour.

Which type of crane should the company choose?

The break-even chart is:
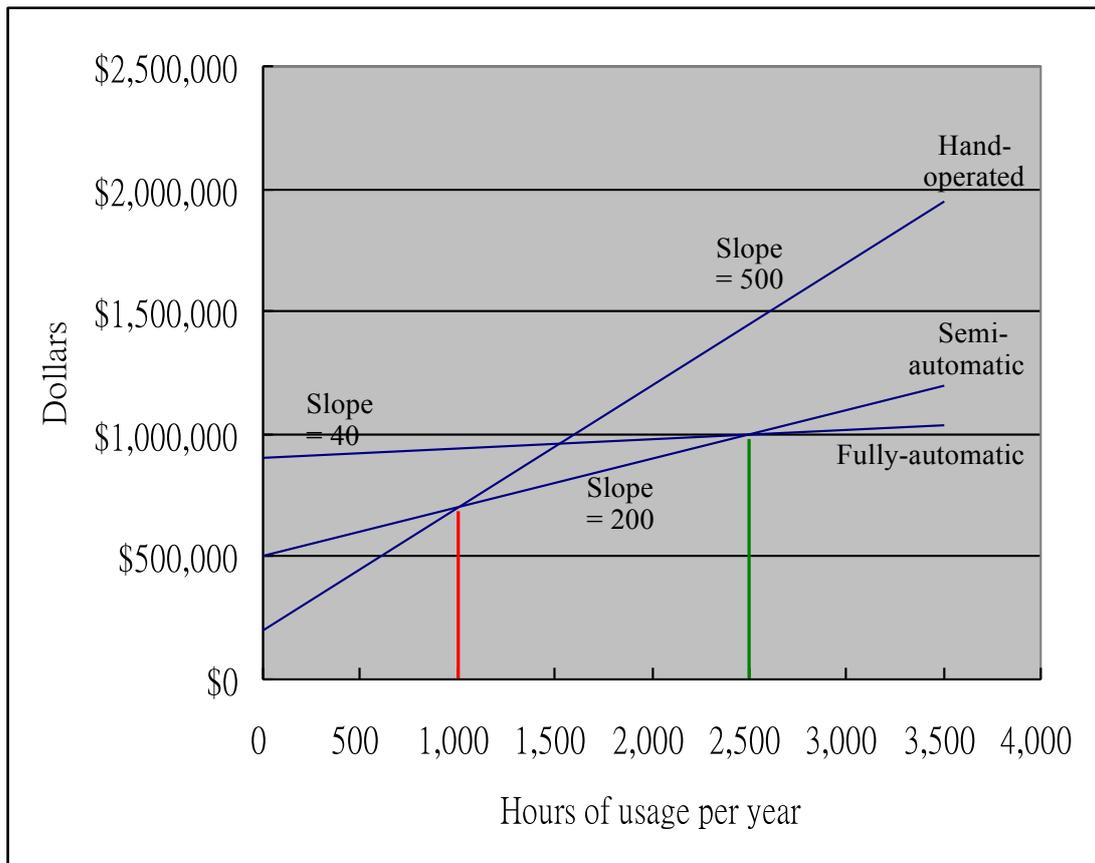
Fig. 6.3 – Break-even chart for the equipment selection decision problem

The break-even chart suggests that the hand operated crane has the lowest total cost up to a volume of 1,000 hours of usage in a year. In the 1,000 hours to 2,500 hours range in a year, the semi-automatic crane has the lowest total cost. At volumes over 2,500 hours of usage in a year, the fully automatic crane has the lowest total cost.

6.8   Engineering Scheme Choice Decision

We are going to see the third example on the application of break-even chart. In this example, formula introduced in Chapter 3 will be used.

Example 6.5

A sewage pumping station is being designed. Three possible pumping schemes are proposed and the itemized costs of each scheme are shown below:

Scheme number		Scheme A	Scheme B	Scheme C
Pump	Cost of pumps ($)	120,000	190,000	285,000
	Life (years)	14	16	20
	Maintenance ($/year)	18,000	16,500	16,000
Pipe	Cost of pipes ($)	200,000	160,000	100,000
	Life (years)	30	30	30
	Cost of pumping ($/hour)	2.00	1.60	1.20

What is the most economical range of pumping time in hours/year for each scheme?

(Take i = 5% p.a. and maximum pumping hours in a year = 8,760 hours)

Answer:

Scheme A:

Equivalent annual cost of installation and maintenance

$$= \$120{,}000 \times \left[\frac{0.05(1+0.05)^{14}}{(1+0.05)^{14}-1} \right] + \$200{,}000 \times \left[\frac{0.05(1+0.05)^{30}}{(1+0.05)^{30}-1} \right] + \$18{,}000$$

$$= \$120{,}000 \times 0.1010 + \$200{,}000 \times 0.0651 + \$18{,}000 = \$43{,}140$$

Scheme B:

Equivalent annual cost of installation and maintenance

$$= \$190{,}000 \times \left[\frac{0.05(1+0.05)^{16}}{(1+0.05)^{16}-1} \right] + \$160{,}000 \times \left[\frac{0.05(1+0.05)^{30}}{(1+0.05)^{30}-1} \right] + \$16{,}500$$

$$= \$190{,}000 \times 0.0923 + \$160{,}000 \times 0.0651 + \$16{,}500 = \$44{,}453$$

Scheme C:

Equivalent annual cost of installation and maintenance

$$= \$285{,}000 \times \left[\frac{0.05(1+0.05)^{20}}{(1+0.05)^{20}-1} \right] + \$100{,}000 \times \left[\frac{0.05(1+0.05)^{30}}{(1+0.05)^{30}-1} \right] + \$16{,}000$$

$$= \$285{,}000 \times 0.0802 + \$100{,}000 \times 0.0651 + \$16{,}000 = \$45{,}367$$

The break-even chart is:
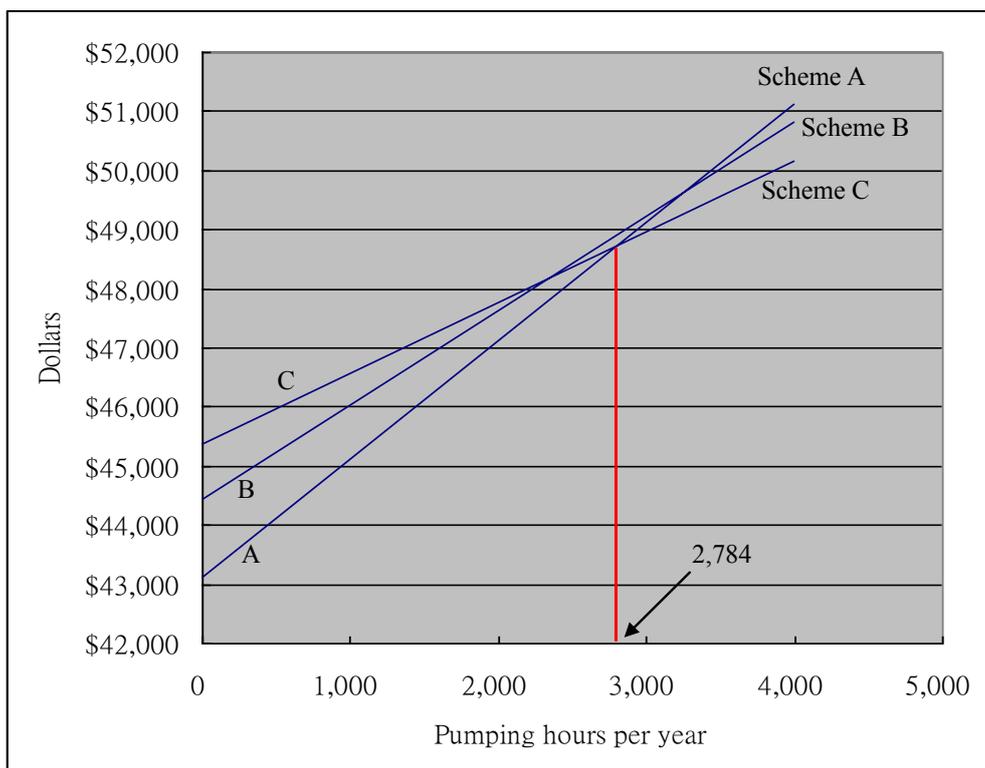
Fig. 6.4 – Break-even chart for the scheme choice decision problem

In Fig. 6.4, the slope of the line for Scheme A is 2, and those for Schemes B and C are 1.6 and 1.2 respectively. The fixed costs for Schemes A, B and C are $43,140, $44,453 and $45,367 respectively. From this break-even chart, we see that for pumping time smaller than 2,784 hours per year, Scheme A is the best. For pumping time larger than 2,784 hours per year, Scheme C is the best. Scheme B is never to be used because it is in no situation better than Scheme A or Scheme C.

6.9   Concluding Remarks

There are limitations in using cost-volume-profit analysis. It is useful only if the construction company has a good cost accounting system and adequate figures, which should be valid, accurate and readily available.

The cost-volume-profit analysis is not a tool for long range planning. Its use should be confined to only a not long duration. The analysis is usually limited to the budget period of a construction company, usually one calendar year.

The analysis presents only a simplified cost, volume and profit relationship. In reality, each is subject to outside influences and to the influences of the other two. The analysis, after all, should be regarded as a help to management decision, but not as a substitute for common sense, logical thinking and human judgment.

Exercise Questions for Chapter 6

Exercise Question 1

A contractor is considering whether he could drop an item and replace it with another from his precast concrete yard. These are his current data:

Precast concrete	p	v	% of total sales
Roof panel	$600	$400	30%
Window panel	$1,000	$600	20%
Seawall panel	$2,000	$1,200	50%

Total fixed cost per year is $7,500,000 and total sales this year is $25,000,000.

The change being considered is to drop the window panel production and replace it with wall panel production with following percentages of total sales:

Precast concrete	p	v	% of total sales
Roof panel	$600	$400	50%
Wall panel	$1,600	$700	10%
Seawall panel	$2,000	$1,200	40%

Total fixed cost per year is $7,500,000 and the estimated total sales next year is $26,00,000

Is this change viable?

Exercise Question 2

A flood control pumping station is being designed. Three schemes are proposed and the itemized costs of each scheme are shown below:

Scheme number		Scheme A	Scheme B	Scheme C
Pump	Cost of pumps ($)	120,000	180,000	280,000
	Life (years)	15	15	20
	Maintenance ($/year)	10,000	15,000	15,000
Pipe	Cost of pipes ($)	220,000	180,000	120,000
	Life (years)	30	30	30
	Cost of pumping ($/hour)	12.00	9.00	8.00

What is the most economical range of pumping time in hours/year for each scheme?

(Take i = 18% p.a. and maximum pumping hours in a year = 24 × 365 = 8,760 hours)

7   Financial Analysis of a Project

7.1   Introduction

In this chapter, we are going to see how we analyze the financial viability of a project. This chapter begins by describing the basic method employed to do financial analysis (financial analysis is different from financial statement analysis as described in Chapter 2). Then a more complicated case study will follow to show the detailed procedures of financial analysis. Readers will learn how to create an income statement and a balance sheet, the two most important financial statements as discussed in Chapters 1 and 2.

Readers should note that the two financial statements (income statement and balance sheet) are meant for the whole company for the examples in Chapters 1 and 2. However, these two financial statements discussed in this chapter are for a project only.

In a financial analysis,

1) depreciation of assets is tax deductible, and
2) interest on loan is also tax deductible.

For an all-equity investment case, there is no loan and hence no interest has to be paid so the only item that is tax deductible is depreciation. This can allow the analysis to be simpler. Example 7.1 in the next section (Section 7.2) is an example of such a case. Example 7.2 in the same section, however, is an equity-plus-loan case, and of course the analysis will be a little more complicated.

7.2   Basic Method of Financial Analysis

Example 7.1 – An all-equity case

A small subcontractor is considering purchasing rock drilling equipment, which has a life of four years and costs $75,000. The average revenue generated per year is estimated to be $40,000 coming from rock drilling related works. The expense associated with maintaining the equipment is estimated to be $15,000 per annum. The profit tax rate is 25%. Assuming straight-line depreciation for the equipment and ignoring salvage value, carry out a financial analysis for the subcontractor.

Answer:

Assuming straight line depreciation and zero salvage value after four years, the annual depreciation of the rock drilling equipment is $18,750 (i.e. 75,000 ÷ 4). An income statement can be produced as shown in Table 7.1.

	Year 1	Year 2	Year 3	Year 4
Revenue	40,000.00	40,000.00	40,000.00	40,000.00
Cost: Expenses	15,000.00	15,000.00	15,000.00	15,000.00
Depreciation	18,750.00	18,750.00	18,750.00	18,750.00
Net Profit before Tax	6,250.00	6,250.00	6,250.00	6,250.00
Tax Expense (25%)	1,562.50	1,562.50	1,562.50	1,562.50
Net Profit after Tax	4,687.50	4,687.50	4,687.50	4,687.50

Table 7.1 – Income statement for financial analysis

The net profit after tax of $4,687.50 per annum shown in the income statement, however, cannot be used to calculate the IRR. It must be adjusted such that depreciation is added back in order to avoid double counting so the DCF method will be correctly used for calculating the IRR (see the last part of Section 4.2 of Chapter 4). This step is shown in Table 7.2.

	Year 1	Year 2	Year 3	Year 4
Revenue	40,000.00	40,000.00	40,000.00	40,000.00
Cost: Expenses	15,000.00	15,000.00	15,000.00	15,000.00
Depreciation	18,750.00	18,750.00	18,750.00	18,750.00
Net Profit before Tax	6,250.00	6,250.00	6,250.00	6,250.00
Tax Expense (25%)	1,562.50	1,562.50	1,562.50	1,562.50
Net Profit after Tax	4,687.50	4,687.50	4,687.50	4,687.50
Add Depreciation	18,750.00	18,750.00	18,750.00	18,750.00
NCF (to find IRR)	23,437.50	23,437.50	23,437.50	23,437.50

Table 7.2 – Cash flows that are used to calculate IRR

The cash flow table for IRR calculation is shown in Table 7.3:

	Cash out	Cash in	
0	75,000.00		
1		23,437.50	
2		23,437.50	IRR = 9.56%, NPV = $8,108.21 (using i = 5% p.a.)
3		23,437.50	
4		23,437.50	

Table 7.3 – All-equity case cash flows

We should note that in Table 7.2 depreciation is deducted first as cost because it is tax deductible. This step allows us to know how much the income tax payable is. In order to avoid double counting of depreciation in using the DCF method, we have to add depreciation back for the calculation of IRR.

Example 7.2 – (Equity + Loan) Case

Repeat doing Example 7.1 if the total capital of $75,000 is made up of $15,000 equity and $60,000 loan from a bank at an interest rate of 5% p.a. for a period of 3 years. The three end of year amortizations for the loan are equal, $20,000 each time. This time the financial analysis has to be modified. Carry out the modified financial analysis.

Answer:

The interest payments for the $60,000 loan are calculated in a way similar to what has been discussed in Section 3.5 of Chapter 3. The calculation is shown in Table 7.4.

End of Year	Amount borrowed	Principal amortization	Balance unamortized	Interest (5%)
0	60,000		60,000	
1		20,000	40,000	3,000
2		20,000	20,000	2,000
3		20,000	0	1,000

Table 7.4 – Interest payments for a loan of $60,000

Therefore, the modified income statement will be as follows:

	Year 1	Year 2	Year 3	Year 4
Revenue	40,000.00	40,000.00	40,000.00	40,000.00
Cost: Expenses	15,000.00	15,000.00	15,000.00	15,000.00
Depreciation	18,750.00	18,750.00	18,750.00	18,750.00
Operating Profit	6,250.00	6,250.00	6,250.00	6,250.00
Interest Expense	3,000.00	2,000.00	1,000.00	-
Net Profit before Tax	3,250.00	4,250.00	5,250.00	6,250.00
Tax Expense (25%)	812.50	1,062.50	1,312.50	1,562.50
Net Profit after Tax	2,437.50	3,187.50	3,937.50	4,687.50
Add Depreciation	18,750.00	18,750.00	18,750.00	18,750.00
NCF before amortization	21,187.50	21,937.50	22,687.50	23,437.50
Less Amortization	20,000.00	20,000.00	20,000.00	-
NCF (to find IRR)	**1,187.50**	**1,937.50**	**2,687.50**	**23,437.50**

Table 7.5 – Cash flows used to calculate IRR

We must note that interest on loan, like depreciation, is tax deductible. Therefore, we can see from Table 7.5 that interest expense has to be deducted first before tax expense is calculated. Amortization is not tax deductible because it is neither a cost of revenue nor an operating expense, and so it is considered only after the tax expense has been dealt with.

The cash flow table for IRR calculation is shown in Table 7.6:

	Cash out	Cash in	
0	15,000.00		
1		1,187.50	
2		1,937.50	IRR = 20.44%, NPV = $9,491.97 (using i = 5% p.a.)
3		2,687.50	
4		23,437.50	

Table 7.6 – (Equity + Loan) Case NCF

As is explained in Section 4.4 of Chapter 4, the IRR in Example 7.2 is more than double of that in Example 7.1. This is because the all-equity IRR (9.56% p.a.) is much higher than the interest rate on loan (5% p.a.). For such a case, the IRR will increase if a loan is borrowed. The NPV, however, remains stable. The NPV in Example 7.1 is about 15% lower than that in Example 7.2. They are not equal to each other (like Example 4.4 of Chapter 4) because the mathematical process has been disturbed by the subtraction of tax expense.

As a remark to Example 7.2, the revenue and cost for each of these four years are constant base year prices (see Sections 5.3 and 5.4 of Chapter 5). However, the interest expenses and the loan amortizations in the first three years are actual transactions (not constant year based). So, in Table 7.2, the numerical figures are a mixture of real and apparent (nominal) values. The real IRR should be even higher than 20.44%, because the interest expenses and the amortizations could be adjusted to some lower values due to the effect of inflation if we want to convert them to constant base year prices.

7.3   A Case Study on Financial Analysis

We are going to see a more complicated example of financial analysis. It is a case study on analyzing the financial viability of a new project. Besides using the basic method as described in Section 7.2 which involves the income statement only, balance sheet and break-even chart are also used in this case study.

7.3.1   The Proposed Project

The project is concerned with a special type of cement for construction use. It was initiated by a group of four professionals (the partners) – two engineers, one marketing specialist and one finance professional. One of the two engineers conducted the estimates of the new plant and equipment costs while the other estimated the cost of making this special type of cement. After completing the market study by the marketing specialist and the technical study by the two engineers, the finance person started the task of evaluating the financial viability.

7.3.2   Summary of Market Study

a) Market share – the marketing specialist studied that the sales of the following number of bags of special cement would be feasible in the next ten years, including the consideration of the operational growth of the project in the first few years:

Target annual sales:

Year 1   = 105,000 units (1 unit = 1 bag of special cement)

Year 2   = 120,000 units

Year 3   = 135,000 units

Years 4 – 10 = 150,000 units

(See Note at the end of this chapter)

b) Price – the estimated price of each unit = $100 would be reasonable and competitive.
c) Selling expenses per unit = $3.33 (e.g. promoting, advertising, etc.)

7.3.3   Summary of Technical Study

a) Cost of producing the cement

    (i)   Variable cost per unit

Raw materials	$20.00
Direct labour	$13.33
Variable overhead	$10.00
Total variable cost per unit =	$43.33

    (ii)   Fixed cost

Depreciation per year	$1,300,000
Fixed overhead per year	$500,000
Total fixed cost=	$1,800,000

b) Capital requirements

    (i)   Plant-proper

Plant and equipment, buildings, etc.	$13,000,000
Land	$2,250,000
Total Plant-proper =	$15,250,000

    (ii)   Working capital

Operating expenses (1 month)	$100,000
Accounts receivable (1 month)	$875,000 (see Endnote 1)
Inventory	
Raw materials (1 month)	$175,000 (see Endnote 2)
Goods-in-process (0.5 month)	$190,000 (see Endnote 3)
Finished goods (1 month)	$380,000 (see Endnote 4)
Gross working capital required	$1,720,000
Less trade credits on raw materials	$175,000 (see Endnote 5)
Required working capital =	$1,545,000

Total capital required = (i) + (ii) = $16,795,000

Assumptions in financial analysis:

1. Accounts receivable = 1 month sales
2. Inventory: raw materials = 1 month usage

    goods-in-process = 0.5 month of finished goods level

    finished goods = 1 month of cost of goods sold

3. Trade credits = 1 month of raw materials purchased
4. Tax rate = 25% of net taxable income
5. Plant, equipment, etc. would follow straight-line depreciation; no depreciation on land
6. Pre-operation period = 1 year
7. All figures in Sections 7.3.2 and 7.3.3 were based on the money value of the year that the project was planned

7.3.4   Pro-forma Income Statement and All-equity IRR

From the above data, the finance person prepared the pro-forma income statement and hence calculated the internal rate of return based on its performance forecasting. The pro-forma income statement was as follows (Table 7.7). All figures were constant base year prices.

Pro-forma Project Income Statement

Year	1	2	3	4	5	6	7	8	9	10
1. Revenue	10,500	12,000	13,500	15,000	15,000	15,000	15,000	15,000	15,000	15,000
2. Cost of Revenue										
Raw materials	2,100	2,400	2,700	3,000	3,000	3,000	3,000	3,000	3,000	3,000
Labour	1,400	1,600	1,800	2,000	2,000	2,000	2,000	2,000	2,000	2,000
Total Cost of Revenue	3,500	4,000	4,500	5,000	5,000	5,000	5,000	5,000	5,000	5,000
3. Gross Profit	7,000	8,000	9,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000
4. Operating Expenses										
4.1 Variable overhead										
Advertising expenses	350	400	450	500	500	500	500	500	500	500
Production variable overhead	1,050	1,200	1,350	1,500	1,500	1,500	1,500	1,500	1,500	1,500
Sub-total	1,400	1,600	1,800	2,000	2,000	2,000	2,000	2,000	2,000	2,000
4.2 Fixed overhead										
Depreciation	1,300	1,300	1,300	1,300	1,300	1,300	1,300	1,300	1,300	1,300
Fixed operating expenses	1,200	1,200	1,200	1,200	1,200	1,200	1,200	1,200	1,200	1,200
Production fixed overhead	500	500	500	500	500	500	500	500	500	500
Sub-total	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000
Total operating expenses	4,400	4,600	4,800	5,000	5,000	5,000	5,000	5,000	5,000	5,000
5. Net Profit before Tax	2,600	3,400	4,200	5,000	5,000	5,000	5,000	5,000	5,000	5,000
6. Tax Expense (25%)	650	850	1,050	1,250	1,250	1,250	1,250	1,250	1,250	1,250
7. Net Profit after Tax	1,950	2,550	3,150	3,750	3,750	3,750	3,750	3,750	3,750	3,750

Table 7.7 – Pro-forma income statement of the proposed project (× $1,000)

As explained in Section 7.2, the finance person added Depreciation to the Net Profit after Tax in order that the cash flows were suitable for calculating the IRR, as shown in Table 7.8.

Year	1	2	3	4	5	6	7	8	9	10
Net Profit after Tax	1,950	2,550	3,150	3,750	3,750	3,750	3,750	3,750	3,750	3,750
Add Depreciation	1,300	1,300	1,300	1,300	1,300	1,300	1,300	1,300	1,300	1,300
NCF (to find IRR)	3,250	2,850	4,450	5,050	5,050	5,050	5,050	5,050	5,050	5,050

Table 7.8 – Cash flows used to calculate the IRR for the proposed Project (× $1,000)

So, the cash flow table for calculating the IRR was as follows (Table 7.9):

End of year	Cash-out	Cash-in
0	16,795	
1		3,250
2		3,850
3		4,450
4		5,050
5		5,050
6		5,050
7		5,050
8		5,050
9		5,050
10		8,845*

Table 7.9 – All-equity cash flows of the proposed project (× $1,000)

* 5,050 + salvage value = 5,050 + (land cost + working capital) = 5,050 + (2,250 + 1,545) = 8,845

The all-equity IRR was found to be 23.48% p.a. It seemed to be a high return investment.

7.3.5   Sources of Financing

These financial statements (7.3.2 through 7.3.4) were sent to a few banks/creditors. All showed interest in providing the four partners with loans because the proposed project looked promising as revealed by the financial statements. From these banks/creditors, the partners chose only two sources of financing. They were (1) a foreign supplier of equipment cum creditor and (2) a local bank, as detailed below.

1) A foreign supplier of equipment cum creditor showed its willingness to supply the project's plant and equipment needs on the following terms:
   a) 15% down payment,
   b) balance payable in 10 equal annual principal amortizations,
   c) interest on the balance of 10% p.a. payable together with amortizations,
   d) secured by a mortgage of plant and equipment.

2) A local bank agreed to lend up to 60% of the value of the land to be purchased for the project under the following terms and conditions:
   a) payable in 3 equal annual principal amortizations,
   b) interest of 12% p.a. on outstanding balance payable coincident with amortizations,
   c) secured by a mortgage on the land.

Besides the above sources (1) and (2), the partners had to pool a total resource of at least $4,395,000 as the owners' capital so that the total capital requirement of $16,795,000 could be satisfied. But the partners actually put $4,450,000 into the project which was more than enough. The equity and loan components were as follows (Table 7.10):

Equity (owners' capital)	$4,450,000	Provided by the partners
Loan from Foreign Supplier	$13,000,000 × 85% = $11,050,000	Loan for 10 years, 10% p.a. interest on the balance
Loan from Local Bank	$2,250,000 × 60% = $1,350,000	Loan for the first 3 years, 12% p.a. on the balance
Total = $16,850,000 > $16,795,000, OK		

Table 7.10 – Sources of financing

7.3.6   Final Income Statement and IRR After Financing

Since interest payments on loans were tax deductible, so the interest payments on each loan had to be known. They were calculated and shown in Table 7.11.

Year	Foreign Supplier Principal Amortization	Balance	Interest (10%)	Local bank Principal Amortization	Balance	Interest (12%)	Total interest	Total yearly amortization
1	1,105	11,050	1,105	450	1,350	162	1,267	1,555
2	1,105	9,945	995	450	900	108	1,103	1,555
3	1,105	8,840	884	450	450	54	938	1,555
4	1,105	7,735	774				774	1,105
5	1,105	6,630	663				663	1,105
6	1,105	5,525	553				553	1,105
7	1,105	4,420	442				442	1,105
8	1,105	3,315	332				332	1,105
9	1,105	2,210	221				221	1,105
10	1,105	1,105	111				111	1,105
	11,050			1,350				

Table 7.11 – Interest payments on loans (× $1,000)

Then, interest payments had to be incorporated into the Final Income Statement, as shown in Table 7.12.

Final Income Statement

Year	1	2	3	4	5	6	7	8	9	10
1. Revenue	10,500	12,000	13,500	15,000	15,000	15,000	15,000	15,000	15,000	15,000
2. Cost of Revenue										
Raw materials	2,100	2,400	2,700	3,000	3,000	3,000	3,000	3,000	3,000	3,000
Labour	1,400	1,600	1,800	2,000	2,000	2,000	2,000	2,000	2,000	2,000
Total Cost of Revenue	3,500	4,000	4,500	5,000	5,000	5,000	5,000	5,000	5,000	5,000
3. Gross Profit	7,000	8,000	9,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000
4. Operating Expenses										
4.1 Variable overhead										
Advertising expenses	350	400	450	500	500	500	500	500	500	500
Production variable overhead	1,050	1,200	1,350	1,500	1,500	1,500	1,500	1,500	1,500	1,500
Sub-total	1,400	1,600	1,800	2,000	2,000	2,000	2,000	2,000	2,000	2,000
4.2 Fixed overhead										
Depreciation	1,300	1,300	1,300	1,300	1,300	1,300	1,300	1,300	1,300	1,300
Fixed operating expenses	1,200	1,200	1,200	1,200	1,200	1,200	1,200	1,200	1,200	1,200
Production fixed overhead	500	500	500	500	500	500	500	500	500	500
Sub-total	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000
Total operating expenses	4,400	4,600	4,800	5,000	5,000	5,000	5,000	5,000	5,000	5,000
5. Operating profit	2,600	3,400	4,200	5,000	5,000	5,000	5,000	5,000	5,000	5,000
6. Interest Expense	1,267	1,103	938	774	663	553	442	332	221	111
7. Net profit before Tax	1,333	2,297	3,262	4,226	4,337	4,447	4,558	4,668	4,779	4,889
8. Tax Expense (25% tax rate)	333	574	816	1,057	1,084	1,112	1,140	1,167	1,195	1,222
9. Net profit after tax	1,000	1,723	2,446	3,169	3,253	3,335	3,418	3,501	3,584	3,667
Source of Fund										
10. Add Depreciation	1,300	1,300	1,300	1,300	1,300	1,300	1,300	1,300	1,300	1,300
11. NCF before Amortization	2,300	3,023	3,746	4,469	4,553	4,635	4,718	4,801	4,884	4,967
12. Less Amortization	1,555	1,555	1,555	1,105	1,105	1,105	1,105	1,105	1,105	1,105
13. NCF (to find IRR)	745	1,468	2,191	3,364	3,448	3,530	3,613	3,696	3,779	3,862
14. Dividend										
% of owners' capital (assumed)	10%	25%	50%	60%	80%	80%	80%	80%	100%	100%
Actual amount	445	1,113	2,225	2,670	3,560	3,560	3,560	3,560	4,450	4,450
15. Balance in Source	300	355	(34)	694	(112)	(30)	53	136	(671)	(588)
16. Retained Earnings	300	655	621	1,315	1,203	1,173	1,226	1,362	691	103

Table 7.12 – The final income statement (× $1,000)

The cash flow table for calculating the final IRR was as follows (Table 7.13):

End of year	Cash-out	Cash-in
0	4,395	
1		745
2		1,468
3		2,191
4		3,364
5		3,448
6		3,530
7		3,613
8		3,696
9		3,779
10		7,657*

Table 7.13 – Equity-Plus-Loan NCF of the proposed project (× $1,000)

* 3,862 + salvage value = 3,862 + (land cost + working capital) = 3,862 + (2,250 + 1,545) = 7,657

The IRR was found to be 44.93% p.a., an extremely profitable project with the said loans coming into the picture. Very good dividends could be seen from Item 14 of Table 7.12.

7.3.7   The Balance Sheet

For the balance sheet of this proposed project, it could be created based on the final income statement. It is shown in Table 7.14. The steps of creating it were as follows:

(Only the steps for Year 1 would be described; steps for other years would be the same)
Step 1: Enter 875,000 against the row Receivables
Step 2: Enter 175,000 against the row Raw materials
Step 3: Enter 190,000 against the row Goods-in-process
Step 4: Enter 380,000 against the row Finished goods
Step 5: Enter 2,250,000 against the row Land
Step 6: Enter 13,000,000 against the row Plant, equipment, buildings
Step 7: Enter -1,300,000 or (1,300,000) against the row Less accumulated depreciation
Step 8: Enter 13,950,000, sum of Fixed Assets (Steps 5 through 7), against Sub-total
Step 9: Enter 175,000 against the row Accounts payable

Balance Sheet

	Pre-operation	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
Current Assets											
Cash	1,600,000	1,755,000	1,655,000	1,151,000	1,390,000	1,473,000	1,638,000	1,886,000	2,217,000	1,741,000	243,000
Receivables -	-	875,000	1,000,000	1,125,000	1,250,000	1,250,000	1,250,000	1,250,000	1,250,000	1,250,000	1,250,000
Inventories											
Raw materials	175,000	175,000	200,000	225,000	250,000	250,000	250,000	250,000	250,000	250,000	250,000
Goods-in-process	-	190,000	215,000	245,000	270,000	270,000	270,000	270,000	270,000	270,000	270,000
Finished goods	-	380,000	430,000	490,000	540,000	540,000	540,000	540,000	540,000	540,000	540,000
Sub-total	1,775,000	3,375,000	3,500,000	3,236,000	3,700,000	3,783,000	3,948,000	4,196,000	4,527,000	4,051,000	2,553,000
Fixed Assets											
Land	2,250,000	2,250,000	2,250,000	2,250,000	2,250,000	2,250,000	2,250,000	2,250,000	2,250,000	2,250,000	2,250,000
Plant, equip, buildings	13,000,000	13,000,000	13,000,000	13,000,000	13,000,000	13,000,000	13,000,000	13,000,000	13,000,000	13,000,000	13,000,000
Less accumu. deprn.	-	(1,300,000)	(2,600,000)	(3,900,000)	(5,200,000)	(6,500,000)	(7,800,000)	(9,100,000)	(10,400,000)	(11,700,000)	(13,000,000)
Sub-total	15,250,000	13,950,000	12,650,000	11,350,000	10,050,000	8,750,000	7,450,000	6,150,000	4,850,000	3,550,000	2,250,000
Total Assets	17,025,000	17,325,000	16,150,000	14,586,000	13,750,000	12,533,000	11,398,000	10,346,000	9,377,000	7,601,000	4,803,000
Current Liabilities											
Accounts payable	175,000	175,000	200,000	225,000	250,000	250,000	250,000	250,000	250,000	250,000	250,000
Current maturity	-	1,555,000	1,555,000	1,555,000	1,105,000	1,105,000	1,105,000	1,105,000	1,105,000	1,105,000	1,105,000
Sub-total	175,000	1,730,000	1,755,000	1,780,000	1,355,000	1,355,000	1,355,000	1,355,000	1,355,000	1,355,000	1,355,000
Long-term Liabilities											
Foreign loan	11,050,000	9,945,000	8,840,000	7,735,000	6,630,000	5,525,000	4,420,000	3,315,000	2,210,000	1,105,000	-
Local loan	1,350,000	900,000	450,000	-	-	-	-	-	-	-	-
Sub-total	12,400,000	10,845,000	9,290,000	7,735,000	6,630,000	5,525,000	4,420,000	3,315,000	2,210,000	1,105,000	-
Total Liabilities	12,575,000	12,575,000	11,045,000	9,515,000	7,985,000	6,880,000	5,775,000	4,670,000	3,565,000	2,460,000	250,000
Equity (i.e. Net Worth)											
Capital stock	4,450,000	4,450,000	4,450,000	4,450,000	4,450,000	4,450,000	4,450,000	4,450,000	4,450,000	4,450,000	4,450,000
Retained earnings	-	300,000	655,000	621,000	1,315,000	1,203,000	1,173,000	1,226,000	1,362,000	691,000	103,000
Total Equity	4,450,000	4,750,000	5,105,000	5,071,000	5,765,000	5,653,000	5,623,000	5,676,000	5,812,000	5,141,000	4,553,000
Equity + Total Liabilities	17,025,000	17,325,000	16,150,000	14,586,000	13,750,000	12,533,000	11,398,000	10,346,000	9,377,000	7,601,000	4,803,000

Table 7.14 – Balance sheet of the proposed project

Step 10: Enter 1,555,000 against the row Current maturity

Step 11: Enter 1,770,000, sum of Current Liabilities (Steps 9 and 10), against Sub-total

Step 12: Enter 9,945,000 against the row Foreign loan

Step 13: Enter 900,000 against the row Local loan

Step 14: Enter 10,845,000, sum of Long-term Liabilities (Steps 12 and 13) against Sub-total

Step 15: Enter 12,575,000, sum of two sub-totals (Steps 11 and 14) against Total Liabilities

Step 16: Enter 4,450,000 against the row Capital stock

Step 17: Enter 300,000 against the row Retained earnings (see Table 7.12)

Step 18: Enter 4,750,000, sum of Equity (Steps 16 and 17) against Total Equity

Step 19: Enter 17,325,000, sum of Equity + Total Liabilities (Steps 18 and 15) at the last row

Step 20: Copy 17,325,000 as Total Assets at the middle of balance sheet (see Equation 1.1)

Step 21: Enter 3,375,000 (i.e. 17,325,000 – 13,950,000) against Sub-total of Current Assets

Step 22: Enter 1,755,000 (i.e. 3,375,000 – 875,000 – 175,000 – 190,000 – 380,000) as Cash

So, the column for Year 1 of the balance sheet had been completed. The same method was applied to all other columns so a complete balance sheet (Table 7.14) could be obtained.

7.3.8 The Break-even Chart

The break-even chart for the 4th year of operation was as shown in Fig. 7.1 next page. It was drawn by referencing to Year 4 column of Table 7.12 (the final income statement). Year 4 was chosen because it was the first year of full (100%) operation. In the break-even chart (Fig. 7.1), the revenue line is represented by $y=100x$. The VC (variable cost) line is represented by $y=46.66x$, because the total variable cost for producing 150,000 units of cement bags was $7,000,000 (see Table 7.12). The FC+VC (fixed cost + variable cost) line is represented by $y=46.66x+1,700,000$ because the fixed cost (excluding depreciation) was $1,700,000 in the 4th year. The next line is including depreciation, and is represented by $y=46.66x+3,000,000$. The next line is, besides VC, FC and depreciation, including interest expense also, and is represented by $y=46.66x+3,774,000$, because the interest expense in the 4th year was $774,000. The next line (the last one) is, besides all, including the tax expense. It is represented by $y=46.66x+4,831,000$ because $1,057,000 of tax was added to the fixed cost.

The revenue line intersects with the other five lines at five different points, including the one at the origin. If all costs (including tax expense) were considered, the breakeven point was at a volume of 90,581 units. Therefore, the BEP was at 60.4% capacity of the proposed project (i.e. 90,581 / 150,000) in the 4th year of operation. Readers may find out the break-even points of other years of operation by themselves.
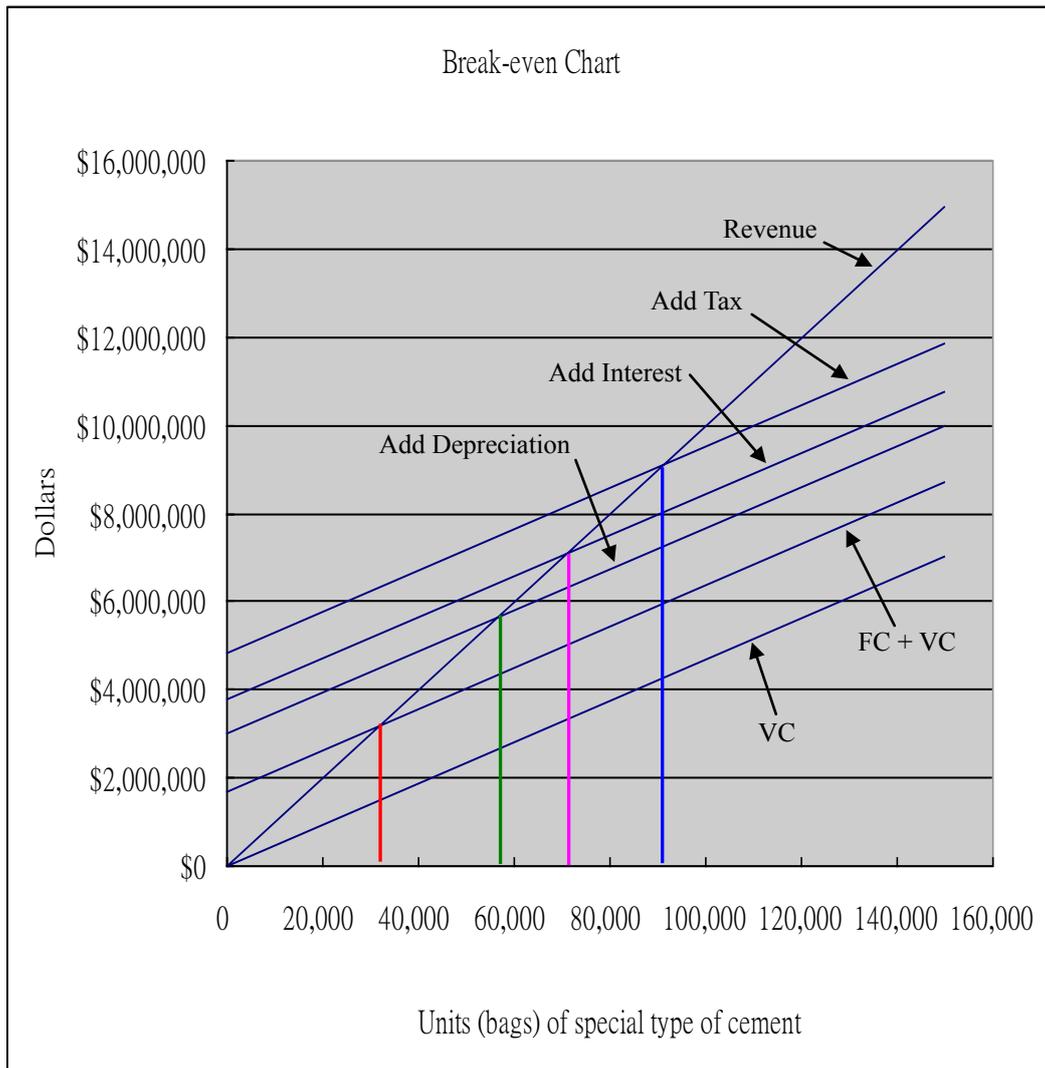
Fig. 7.1 – The break-even chart for the 4th year of operation

Note: For a medium scale project, the time horizon used for financial analysis is usually ten years (i.e. n = 10 years). For small scale projects, n is taken as 4 or 5 years. For large scale projects, n is taken as 25 or 30 years. For huge infrastructure projects, n can be taken as long as 50 or 60 years.

The project described in Section 7.3 is considered as medium scale and so n is taken as 10 years.

Exercise Questions for Chapter 7

Exercise Question 1

A construction company is considering setting up a new precast concrete yard. To do so, a new equipment, which has a life of four years and costs $1,000,000 has to be purchased. The revenue generated is estimated to be $500,000 per year. The total expenses associated with this precast concrete yard are estimated to be $120,000 per year. The profit tax rate is 25%.

a) Assuming straight line depreciation and ignoring salvage values, carry out a financial analysis for the company. (You are expected to create an income statement and derive a cash flow table for IRR calculation, and then to compute the IRR).

b) Carry out a new financial analysis and find the new IRR if the total capital of $1,000,000 is made up of $250,000 equity and $750,000 loan. The loan has to be paid back in three years (i.e. principal amortization of $250,000 per year) at an interest rate of 8% p.a.

c) Compare the internal rates of return in (a) and (b) above and give comments on them. If the NPV method at a discount rate of 8% p.a. (the borrowing interest rate) is used in (a) and (b), what results will be expected? And why?

8 S-Curve and Working Capital Financing of a Construction Project

8.1 Introduction

This chapter discusses what an S-Curve is and how it is derived. In brief, an S-Curve is the graph/ curve showing the cumulative expenditure of a project (against time) spent by a contractor from the very beginning up to the very end of the project during construction. An S-Curve is usually considered together with another graph called cumulative interim payment graph, which shows the cumulative payment paid to the contractor by the owner/client of the project.

In the following sections, we will see how an S-Curve is derived. We will then see how a cumulative interim payment graph is drawn. By combining the two graphs, we will also discuss the maximum working capital required of the contractor for the project and the cost of its lock-up working capital, which, in other words, is the amount of interest, charged to the contractor for the lock-up working capital due to the project, if it is financed by a bank.

8.2 S-Curve

A construction project always has a programme chart (or called bar chart or programme of work) such that each and every activity (or task) in the project is arranged to be carried out at proper time and in appropriate order. The programme/bar chart is usually first derived from a critical path analysis (the discussion of which is outside the scope of this book), and then an adjustment to the chart is made by experienced professionals in the construction company so that the programme of work is finalized in the most realistic manner.

The upper half of Fig. 8.1 (see next page) is an example of a bar chart for a small construction project. We can see a curve at the lower half of Fig. 8.1, and this curve is derived from the bar chart above it using the same time scale (or same x-axis). The cost of each task is given in the bar chart, and the curve below is drawn based on the cumulative cost (or expenditure). The shape of the curve looks like the letter S, and hence we usually call it an S-Curve. From experience, for almost all construction projects, their cumulative expenditure curves look like an S shape, quite similar to the one in Fig. 8.1. This is because at the beginning of a contract, works are catching up; at the middle, works are running smoothly; towards the end, works are slowing down.
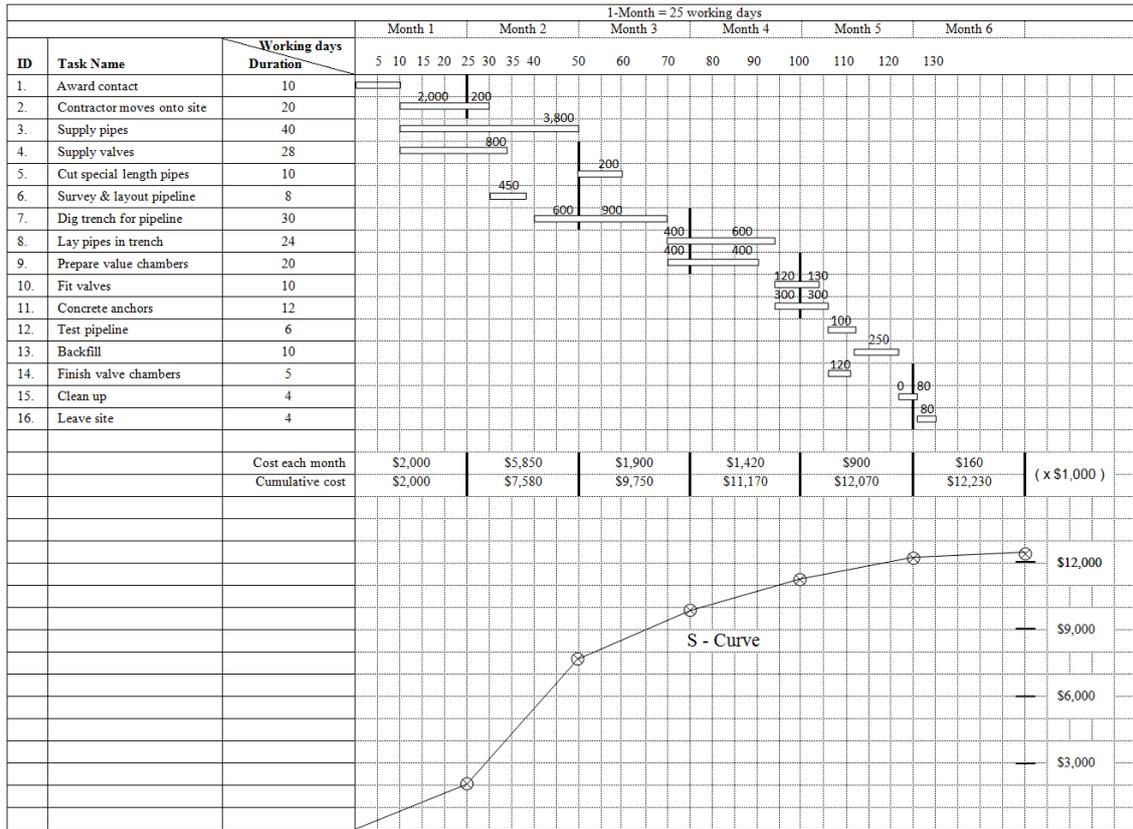
Figure 8.1 – Example of "S Curve" derived from the programme of work of a small construction project

8.3 Cumulative Interim Payment Graph

For construction contracts, payments made by the owner/client to the contractor are on interim basis, that is, payments are made periodically. The length of the period depends on the type of construction contract employed. For a traditional unit rate contract, the interim payments are usually made on monthly basis. For a design-and-build lump sum contract, the interim payments are usually made based on milestones achieved by the contractor that have been agreed with the client at the time of contract signing. In this case, the time periods between any two interim payments can be weeks or months. Supposing that the project in Fig. 8.1 is a traditional unit rate contract with a contract sum of $13,200,000 and that the interim payments are made by the client to the contractor on a monthly basis, Table 8.1 below shows the interim payments made by the client.

End of month	2	3	4	5	6	7	8
Revenue (x $1,000)	3,000	5,000	2,500	1,350	1,020	0	330
Cumulative revenue (x $1,000)	3,000	8,000	10,500	11,850	12,870	12,870	13,200

Table 8.1 – Interim payments from the client to the contractor

The cumulative interim payment graph is a step curve as shown in Fig. 8.2. It reflects the time and the amount of money the contractor receives from the client cumulatively from the very beginning to the very end of the contract. According to Fig. 8.1, the contract completion date is 5 months and 5 days, and the last few days are for "clean up and leave site". In order to make the graph simple, we assume that the contract is substantially completed in 5 months. The maintenance period (or defect liability period) is assumed to be 3 months, and so the retention money (2.5% of the contract sum) can be released at the end of month 8.
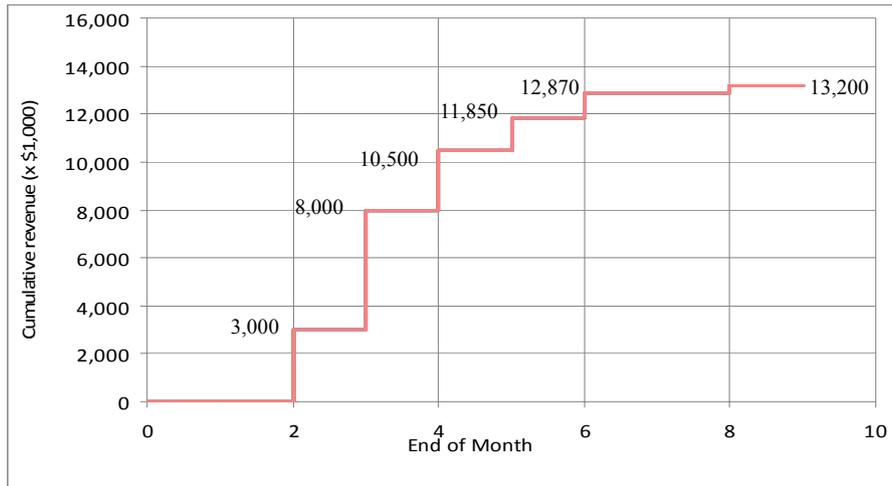
Fig. 8.2 – Cumulative interim payment graph

8.4   The Two Graphs combined

When the S-Curve (the same curve as in Fig. 8.1) and the cumulative interim payment graph are combined to be shown in one diagram, Fig. 8.3 shows how the diagram looks.
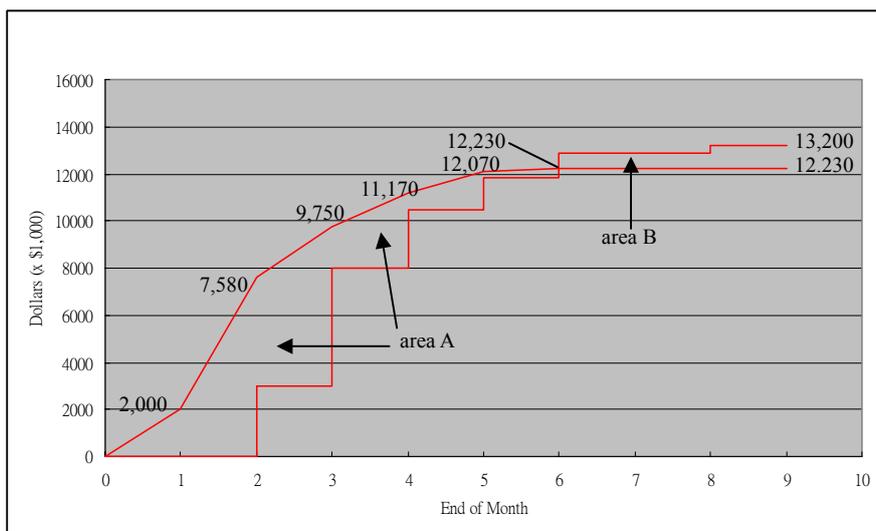
Fig. 8.3 – The S-Curve and the cumulative interim payment graph combined

From the combined graphs, we can find out the maximum working capital required of the contractor for this project, and the amount of interest charged to the contractor for the working capital if it is financed by a bank.

The maximum working capital needed by the contractor can be found from measuring the vertical differences between the two graphs at several points, and then these vertical differences are compared such that the largest difference is identified. This largest difference is the maximum working capital required. In the above example (Fig. 8.3), the difference immediately before the end of month 2 is 7,580 (i.e. 7,580 – 0); the difference immediately before the end of month 3 is 6,750 (i.e. 9,750 – 3,000); the difference immediately before the end of month 4 is 3,170 (i.e. 11,170 – 8,000), and so on.

So, the largest vertical difference occurs immediately before the end of month 2. The maximum amount of working capital required is $7,580,000 for the project.

Apparently, the profit is $970,000 (i.e. 13,200,000 – $12,230,000). However, the contractor does not earn such an amount of profit but less than that. The reason is that the working capital has been locked up by the project, and the cost of the lock-up working capital must be deducted from $970,000, and what remains will then be the true profit. This is related to the so called opportunity cost, which means that if the working capital is not used here, it would have been used elsewhere to make money or gain profits. One way to find out this opportunity cost is to calculate the interest charged to the contractor as if the lock-up working capital is financed by a bank.

To calculate the interest charged (or the cost of the lock-up working capital), we have to find out area A and area B (see Fig. 8.3), the areas between the two curves. They are called finance areas. A finance area is positive if the S-Curve is above the cumulative interim payment curve, and vice versa. Therefore, in this case, finance area A is positive and finance area B is negative. The unit (or dimension) for these areas is $-month. The total (or net) finance area is equal to (area A – area B), calculated up to the end of month 8 only, which, if multiplied by the bank interest rate (in % per month, or % p.a. divided by 12), is the cost of the working capital. If the result of this multiplication is subtracted from $970,000 then the true profit can be known. The worked example in the next section (8.5) will illustrate this.

8.5 A Worked Example

Table 8.2 below shows the contractor's budgeted cost (expenditure) for a 9-month contract.

End of Month	1	2	3	4	5	6	7	8	9
Expenditure in the month (million$)	3 m	4 m	5 m	6 m	7 m	7 m	6 m	4 m	2 m

Table 8.2 – Expenditure each month for a 9-month contract

There is a mark-up of 8% to all unit rates contained in the contract document. The interim payments are made monthly. Due to the availability of trade credits, the contractor can delay paying his expenditures by an average of one month. Therefore, unlike Fig. 8.1, the S-Curve in this example starts at the end of month 1.

At the end of each month, quantity measurements are made, and the interim payment certified less 5% retention money is paid one month later. The accumulated retention money is released 6 months after the date of contract completion. The contract takes 9 months to complete and the contract sum is $47,520,000.

If the interest rate on money borrowing is 6% p.a., what are

a) the maximum amount of working capital required to execute the contract?
b) The cost of working capital for this project?
c) The contractor's true profit gained from this project?

Answer:

The calculation of the cumulative expenditure (S-Curve) and the cumulative interim payments are shown in Table 8.3.

End of Month	1	2	3	4	5	6	7	8	9	10	15
Expenditure (x $1 m)	3	4	5	6	7	7	6	4	2	0	0
Cumulative expenditure (x $1 m)	3	7	12	18	25	32	38	42	44	44	44
S-Curve (x $1 m) (1-month delay)	0	3	7	12	18	25	32	38	42	44	44
Cumulative mark-up (8%)	0.24	0.56	0.96	1.44	2.00	2.56	3.04	3.36	3.52	3.52	3.52
Cumulative value of work (x $1 m)	3.24	7.56	12.96	19.44	27.00	34.56	41.04	45.36	47.52	47.52	47.52
Cumulative value less 5% retention	3.08	7.18	12.31	18.47	25.65	32.83	38.99	43.09	45.144	45.144	45.144
Cumulative interim Payment (x $1 m)	0	3.08	7.18	12.31	18.47	25.65	32.83	38.99	43.09	45.144	47.52

Table 8.3 – Cumulative expenditure and cumulative interim payment against time

The two curves are drawn in Fig. 8.4 as follows.
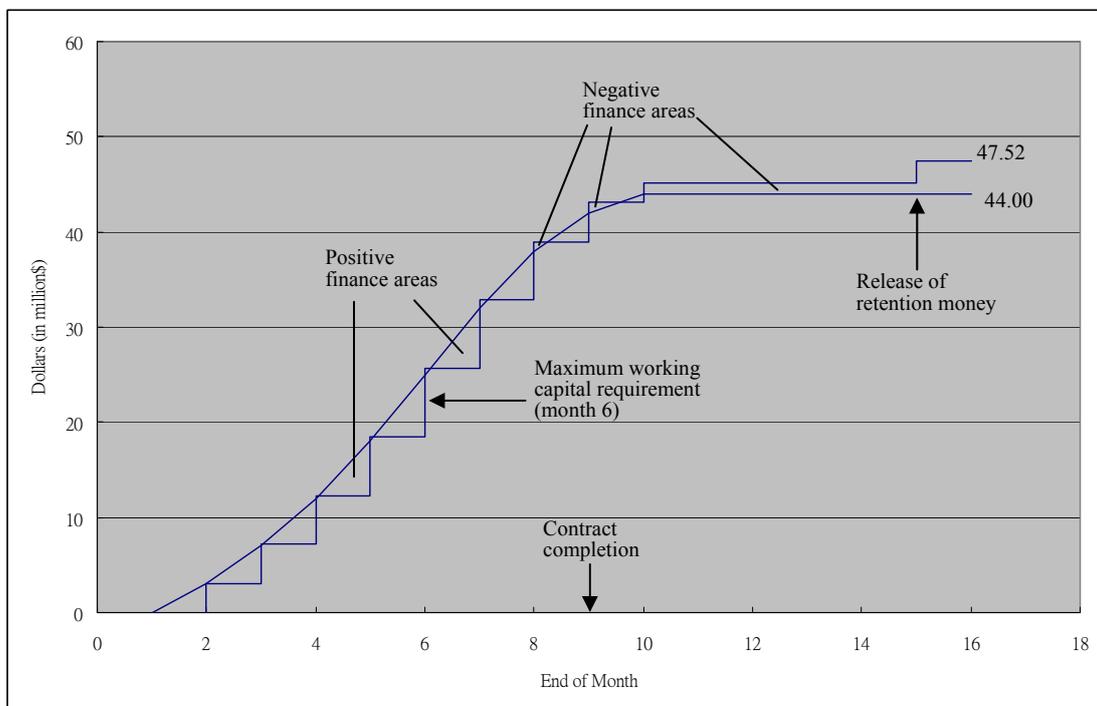
Fig. 8.4 – S-Curve and the cumulative interim payment graph of the worked example

a) By inspection, the maximum vertical difference of the two curves occurs at the time immediately before month 6. Therefore, the maximum working capital required is $6.53m (i.e. 25.00 – 18.47).

b) From Fig.8.4, the positive finance area is found to be approximately 18.6 million$-month, and the negative finance area (up to the end of month 15) approximately 6.2 million$-month. Therefore, the total (or net) finance area is 12.4 million$-month.

   Since the interest rate is 6% p.a. (i.e. 0.5% per month), the interest charges therefore is $0.062 millon (i.e. 12.4 × 0.5%), or $62,000. This $62,000 is also called the cost of working capital for this project.

c) The contractor's true profit gained from this project

   = ($47,520,000 – $44,000,000) – $62,000

   = $3,458,000

   which is about 7.3% of the contract sum.

Exercise Questions for Chapter 8

Question 1

The following table lists the cumulative monthly costs incurred by a contractor and the corresponding monthly payments which are received from the owner of a project.

End of month	Cumulative expenditure	Cumulative revenue
0	0	0
1	120,000	0
2	200,000	0
3	540,000	0
4	900,000	140,000
5	1,300,000	400,000
6	1,800,000	1,000,000
7	2,200,000	1,300,000
8	2,500,000	1,900,000
9	2,700,000	2,100,000
10	2,900,000	3,000,000
11	2,900,000	3,200,000
12	2,900,000	3,400,000

Calculate the cost to the contractor of borrowing the working capital necessary to finance the project if the annual rate of interest he has to pay is 12%.

Question 2

The expenditure per month for a small-medium scale building project scheduled for 5 months are given below. The mark-up is 5%. The retention money of the billed cost is 10% if the cumulative payment is less than half of the total value of work (contract value) and then 0% thereafter. The retention money will be released at the end of month 6, one month after the completion of the contract. Payment is billed at the end of each month and received one month later by the contractor. The finance charge is 1% per month.

End of month	1	2	3	4	5
Expenditure ($)	1,000,000	2,100,000	2,500,000	2,500,000	2,500,000

By plotting the S-Curve and the cumulative interim payment graph, find the peak working capital requirement and the total financial charge on the working capital if it is totally borrowed.

Question 3

The following table shows the value of work a contractor completes and the profit distribution for a 10-month construction contract.

End of month	1	2	3	4	5	6	7	8	9	10
Value of work (million $)	2	3	4	7	7	7	6	5	4	1
Profit (% of value of work)	15	15	10	10	10	10	10	10	5	5

Interim measurements are made monthly and payments of the amount certified less 10% retention are paid the contractor one month later. 50% of the retention money is released one month after substantial completion (i.e. end of month 11, since completion is at end of month 10), and the other 50% retention is released 6 months later (the defect liability period is 6 months).

However, the client desires to decrease his administration duties and proposes to the contractor that interim payments be made bi-monthly. The contractor wishes to know the financial implications of this proposal. In other words, he wants to know how much more he has to pay for the cost of the working capital.

Assuming the contractor can delay paying his expenditures by an average of one month due to the availability of trade credits, calculate the extra financial charge for borrowing the working capital if interest rate is at 9% p.a. and interim payments are made on a bi-monthly basis.

9   Appendix

	$(1+i)^n$	$\dfrac{1}{(1+i)^n}$	$\dfrac{(1+i)^n-1}{i}$	$\dfrac{i}{(1+i)^n-1}$	$\dfrac{(1+i)^n-1}{i(1+i)^n}$	$\dfrac{i(1+i)^n}{(1+i)^n-1}$
n			***i* = 1%**			
1	1.0100	0.9901	1.0000	1.0000	0.9901	1.0100
2	1.0201	0.9803	2.0100	0.4975	1.9704	0.5075
3	1.0303	0.9706	3.0301	0.3300	2.9410	0.3400
4	1.0406	0.9610	4.0604	0.2463	3.9020	0.2563
5	1.0510	0.9515	5.1010	0.1960	4.8534	0.2060
6	1.0615	0.9420	6.1520	0.1625	5.7955	0.1725
7	1.0721	0.9327	7.2135	0.1386	6.7282	0.1486
8	1.0829	0.9235	8.2857	0.1207	7.6517	0.1307
9	1.0937	0.9143	9.3685	0.1067	8.5660	0.1167
10	1.1046	0.9053	10.4622	0.0956	9.4713	0.1056
11	1.1157	0.8963	11.5668	0.0865	10.3676	0.0965
12	1.1268	0.8874	12.6825	0.0788	11.2551	0.0888
13	1.1381	0.8787	13.8093	0.0724	12.1337	0.0824
14	1.1495	0.8700	14.9474	0.0669	13.0037	0.0769
15	1.1610	0.8613	16.0969	0.0621	13.8651	0.0721
16	1.1726	0.8528	17.2579	0.0579	14.7179	0.0679
17	1.1843	0.8444	18.4304	0.0543	15.5623	0.0643
18	1.1961	0.8360	19.6147	0.0510	16.3983	0.0610
19	1.2081	0.8277	20.8109	0.0481	17.2260	0.0581
20	1.2202	0.8195	22.0190	0.0454	18.0456	0.0554
21	1.2324	0.8114	23.2392	0.0430	18.8570	0.0530
22	1.2447	0.8034	24.4716	0.0409	19.6604	0.0509
23	1.2572	0.7954	25.7163	0.0389	20.4558	0.0489
24	1.2697	0.7876	26.9735	0.0371	21.2434	0.0471
25	1.2824	0.7798	28.2432	0.0354	22.0232	0.0454
n			***i* = 2%**			
1	1.0200	0.9804	1.0000	1.0000	0.9804	1.0200
2	1.0404	0.9612	2.0200	0.4950	1.9416	0.5150
3	1.0612	0.9423	3.0604	0.3268	2.8839	0.3468
4	1.0824	0.9238	4.1216	0.2426	3.8077	0.2626
5	1.1041	0.9057	5.2040	0.1922	4.7135	0.2122
6	1.1262	0.8880	6.3081	0.1585	5.6014	0.1785
7	1.1487	0.8706	7.4343	0.1345	6.4720	0.1545
8	1.1717	0.8535	8.5830	0.1165	7.3255	0.1365
9	1.1951	0.8368	9.7546	0.1025	8.1622	0.1225
10	1.2190	0.8203	10.9497	0.0913	8.9826	0.1113
11	1.2434	0.8043	12.1687	0.0822	9.7868	0.1022
12	1.2682	0.7885	13.4121	0.0746	10.5753	0.0946
13	1.2936	0.7730	14.6803	0.0681	11.3484	0.0881
14	1.3195	0.7579	15.9739	0.0626	12.1062	0.0826
15	1.3459	0.7430	17.2934	0.0578	12.8493	0.0778
16	1.3728	0.7284	18.6393	0.0537	13.5777	0.0737
17	1.4002	0.7142	20.0121	0.0500	14.2919	0.0700
18	1.4282	0.7002	21.4123	0.0467	14.9920	0.0667
19	1.4568	0.6864	22.8406	0.0438	15.6785	0.0638
20	1.4859	0.6730	24.2974	0.0412	16.3514	0.0612
21	1.5157	0.6598	25.7833	0.0388	17.0112	0.0588
22	1.5460	0.6468	27.2990	0.0366	17.6580	0.0566
23	1.5769	0.6342	28.8450	0.0347	18.2922	0.0547
24	1.6084	0.6217	30.4219	0.0329	18.9139	0.0529
25	1.6406	0.6095	32.0303	0.0312	19.5235	0.0512

	$(1+i)^n$	$\dfrac{1}{(1+i)^n}$	$\dfrac{(1+i)^n-1}{i}$	$\dfrac{i}{(1+i)^n-1}$	$\dfrac{(1+i)^n-1}{i(1+i)^n}$	$\dfrac{i(1+i)^n}{(1+i)^n-1}$
n			$i=3\%$			
1	1.0300	0.9709	1.0000	1.0000	0.9709	1.0300
2	1.0609	0.9426	2.0300	0.4926	1.9135	0.5226
3	1.0927	0.9151	3.0909	0.3235	2.8286	0.3535
4	1.1255	0.8885	4.1836	0.2390	3.7171	0.2690
5	1.1593	0.8626	5.3091	0.1884	4.5797	0.2184
6	1.1941	0.8375	6.4684	0.1546	5.4172	0.1846
7	1.2299	0.8131	7.6625	0.1305	6.2303	0.1605
8	1.2668	0.7894	8.8923	0.1125	7.0197	0.1425
9	1.3048	0.7664	10.1591	0.0984	7.7861	0.1284
10	1.3439	0.7441	11.4639	0.0872	8.5302	0.1172
11	1.3842	0.7224	12.8078	0.0781	9.2526	0.1081
12	1.4258	0.7014	14.1920	0.0705	9.9540	0.1005
13	1.4685	0.6810	15.6178	0.0640	10.6350	0.0940
14	1.5126	0.6611	17.0863	0.0585	11.2961	0.0885
15	1.5580	0.6419	18.5989	0.0538	11.9379	0.0838
16	1.6047	0.6232	20.1569	0.0496	12.5611	0.0796
17	1.6528	0.6050	21.7616	0.0460	13.1661	0.0760
18	1.7024	0.5874	23.4144	0.0427	13.7535	0.0727
19	1.7535	0.5703	25.1169	0.0398	14.3238	0.0698
20	1.8061	0.5537	26.8704	0.0372	14.8775	0.0672
21	1.8603	0.5375	28.6765	0.0349	15.4150	0.0649
22	1.9161	0.5219	30.5368	0.0327	15.9369	0.0627
23	1.9736	0.5067	32.4529	0.0308	16.4436	0.0608
24	2.0328	0.4919	34.4265	0.0290	16.9355	0.0590
25	2.0938	0.4776	36.4593	0.0274	17.4131	0.0574
n			$i=4\%$			
1	1.0400	0.9615	1.0000	1.0000	0.9615	1.0400
2	1.0816	0.9246	2.0400	0.4902	1.8861	0.5302
3	1.1249	0.8890	3.1216	0.3203	2.7751	0.3603
4	1.1699	0.8548	4.2465	0.2355	3.6299	0.2755
5	1.2167	0.8219	5.4163	0.1846	4.4518	0.2246
6	1.2653	0.7903	6.6330	0.1508	5.2421	0.1908
7	1.3159	0.7599	7.8983	0.1266	6.0021	0.1666
8	1.3686	0.7307	9.2142	0.1085	6.7327	0.1485
9	1.4233	0.7026	10.5828	0.0945	7.4353	0.1345
10	1.4802	0.6756	12.0061	0.0833	8.1109	0.1233
11	1.5395	0.6496	13.4864	0.0741	8.7605	0.1141
12	1.6010	0.6246	15.0258	0.0666	9.3851	0.1066
13	1.6651	0.6006	16.6268	0.0601	9.9856	0.1001
14	1.7317	0.5775	18.2919	0.0547	10.5631	0.0947
15	1.8009	0.5553	20.0236	0.0499	11.1184	0.0899
16	1.8730	0.5339	21.8245	0.0458	11.6523	0.0858
17	1.9479	0.5134	23.6975	0.0422	12.1657	0.0822
18	2.0258	0.4936	25.6454	0.0390	12.6593	0.0790
19	2.1068	0.4746	27.6712	0.0361	13.1339	0.0761
20	2.1911	0.4564	29.7781	0.0336	13.5903	0.0736
21	2.2788	0.4388	31.9692	0.0313	14.0292	0.0713
22	2.3699	0.4220	34.2480	0.0292	14.4511	0.0692
23	2.4647	0.4057	36.6179	0.0273	14.8568	0.0673
24	2.5633	0.3901	39.0826	0.0256	15.2470	0.0656
25	2.6658	0.3751	41.6459	0.0240	15.6221	0.0640

	$(1+i)^n$	$\dfrac{1}{(1+i)^n}$	$\dfrac{(1+i)^n-1}{i}$	$\dfrac{i}{(1+i)^n-1}$	$\dfrac{(1+i)^n-1}{i(1+i)^n}$	$\dfrac{i(1+i)^n}{(1+i)^n-1}$
n			**i = 5%**			
1	1.0500	0.9524	1.0000	1.0000	0.9524	1.0500
2	1.1025	0.9070	2.0500	0.4878	1.8594	0.5378
3	1.1576	0.8638	3.1525	0.3172	2.7232	0.3672
4	1.2155	0.8227	4.3101	0.2320	3.5460	0.2820
5	1.2763	0.7835	5.5256	0.1810	4.3295	0.2310
6	1.3401	0.7462	6.8019	0.1470	5.0757	0.1970
7	1.4071	0.7107	8.1420	0.1228	5.7864	0.1728
8	1.4775	0.6768	9.5491	0.1047	6.4632	0.1547
9	1.5513	0.6446	11.0266	0.0907	7.1078	0.1407
10	1.6289	0.6139	12.5779	0.0795	7.7217	0.1295
11	1.7103	0.5847	14.2068	0.0704	8.3064	0.1204
12	1.7959	0.5568	15.9171	0.0628	8.8633	0.1128
13	1.8856	0.5303	17.7130	0.0565	9.3936	0.1065
14	1.9799	0.5051	19.5986	0.0510	9.8986	0.1010
15	2.0789	0.4810	21.5786	0.0463	10.3797	0.0963
16	2.1829	0.4581	23.6575	0.0423	10.8378	0.0923
17	2.2920	0.4363	25.8404	0.0387	11.2741	0.0887
18	2.4066	0.4155	28.1324	0.0355	11.6896	0.0855
19	2.5270	0.3957	30.5390	0.0327	12.0853	0.0827
20	2.6533	0.3769	33.0660	0.0302	12.4622	0.0802
21	2.7860	0.3589	35.7193	0.0280	12.8212	0.0780
22	2.9253	0.3418	38.5052	0.0260	13.1630	0.0760
23	3.0715	0.3256	41.4305	0.0241	13.4886	0.0741
24	3.2251	0.3101	44.5020	0.0225	13.7986	0.0725
25	3.3864	0.2953	47.7271	0.0210	14.0939	0.0710
n			**i = 6%**			
1	1.0600	0.9434	1.0000	1.0000	0.9434	1.0600
2	1.1236	0.8900	2.0600	0.4854	1.8334	0.5454
3	1.1910	0.8396	3.1836	0.3141	2.6730	0.3741
4	1.2625	0.7921	4.3746	0.2286	3.4651	0.2886
5	1.3382	0.7473	5.6371	0.1774	4.2124	0.2374
6	1.4185	0.7050	6.9753	0.1434	4.9173	0.2034
7	1.5036	0.6651	8.3938	0.1191	5.5824	0.1791
8	1.5938	0.6274	9.8975	0.1010	6.2098	0.1610
9	1.6895	0.5919	11.4913	0.0870	6.8017	0.1470
10	1.7908	0.5584	13.1808	0.0759	7.3601	0.1359
11	1.8983	0.5268	14.9716	0.0668	7.8869	0.1268
12	2.0122	0.4970	16.8699	0.0593	8.3838	0.1193
13	2.1329	0.4688	18.8821	0.0530	8.8527	0.1130
14	2.2609	0.4423	21.0151	0.0476	9.2950	0.1076
15	2.3966	0.4173	23.2760	0.0430	9.7122	0.1030
16	2.5404	0.3936	25.6725	0.0390	10.1059	0.0990
17	2.6928	0.3714	28.2129	0.0354	10.4773	0.0954
18	2.8543	0.3503	30.9057	0.0324	10.8276	0.0924
19	3.0256	0.3305	33.7600	0.0296	11.1581	0.0896
20	3.2071	0.3118	36.7856	0.0272	11.4699	0.0872
21	3.3996	0.2942	39.9927	0.0250	11.7641	0.0850
22	3.6035	0.2775	43.3923	0.0230	12.0416	0.0830
23	3.8197	0.2618	46.9958	0.0213	12.3034	0.0813
24	4.0489	0.2470	50.8156	0.0197	12.5504	0.0797
25	4.2919	0.2330	54.8645	0.0182	12.7834	0.0782

$(1+i)^n$	$\dfrac{1}{(1+i)^n}$	$\dfrac{(1+i)^n-1}{i}$	$\dfrac{i}{(1+i)^n-1}$	$\dfrac{(1+i)^n-1}{i(1+i)^n}$	$\dfrac{i(1+i)^n}{(1+i)^n-1}$
n			**i = 7%**		
1	1.0700	0.9346	1.0000	1.0000	0.9346	1.0700
2	1.1449	0.8734	2.0700	0.4831	1.8080	0.5531
3	1.2250	0.8163	3.2149	0.3111	2.6243	0.3811
4	1.3108	0.7629	4.4399	0.2252	3.3872	0.2952
5	1.4026	0.7130	5.7507	0.1739	4.1002	0.2439
6	1.5007	0.6663	7.1533	0.1398	4.7665	0.2098
7	1.6058	0.6227	8.6540	0.1156	5.3893	0.1856
8	1.7182	0.5820	10.2598	0.0975	5.9713	0.1675
9	1.8385	0.5439	11.9780	0.0835	6.5152	0.1535
10	1.9672	0.5083	13.8164	0.0724	7.0236	0.1424
11	2.1049	0.4751	15.7836	0.0634	7.4987	0.1334
12	2.2522	0.4440	17.8885	0.0559	7.9427	0.1259
13	2.4098	0.4150	20.1406	0.0497	8.3577	0.1197
14	2.5785	0.3878	22.5505	0.0443	8.7455	0.1143
15	2.7590	0.3624	25.1290	0.0398	9.1079	0.1098
16	2.9522	0.3387	27.8881	0.0359	9.4466	0.1059
17	3.1588	0.3166	30.8402	0.0324	9.7632	0.1024
18	3.3799	0.2959	33.9990	0.0294	10.0591	0.0994
19	3.6165	0.2765	37.3790	0.0268	10.3356	0.0968
20	3.8697	0.2584	40.9955	0.0244	10.5940	0.0944
21	4.1406	0.2415	44.8652	0.0223	10.8355	0.0923
22	4.4304	0.2257	49.0057	0.0204	11.0612	0.0904
23	4.7405	0.2109	53.4361	0.0187	11.2722	0.0887
24	5.0724	0.1971	58.1767	0.0172	11.4693	0.0872
25	5.4274	0.1842	63.2490	0.0158	11.6536	0.0858
n			**i = 8%**		
1	1.0800	0.9259	1.0000	1.0000	0.9259	1.0800
2	1.1664	0.8573	2.0800	0.4808	1.7833	0.5608
3	1.2597	0.7938	3.2464	0.3080	2.5771	0.3880
4	1.3605	0.7350	4.5061	0.2219	3.3121	0.3019
5	1.4693	0.6806	5.8666	0.1705	3.9927	0.2505
6	1.5869	0.6302	7.3359	0.1363	4.6229	0.2163
7	1.7138	0.5835	8.9228	0.1121	5.2064	0.1921
8	1.8509	0.5403	10.6366	0.0940	5.7466	0.1740
9	1.9990	0.5002	12.4876	0.0801	6.2469	0.1601
10	2.1589	0.4632	14.4866	0.0690	6.7101	0.1490
11	2.3316	0.4289	16.6455	0.0601	7.1390	0.1401
12	2.5182	0.3971	18.9771	0.0527	7.5361	0.1327
13	2.7196	0.3677	21.4953	0.0465	7.9038	0.1265
14	2.9372	0.3405	24.2149	0.0413	8.2442	0.1213
15	3.1722	0.3152	27.1521	0.0368	8.5595	0.1168
16	3.4259	0.2919	30.3243	0.0330	8.8514	0.1130
17	3.7000	0.2703	33.7502	0.0296	9.1216	0.1096
18	3.9960	0.2502	37.4502	0.0267	9.3719	0.1067
19	4.3157	0.2317	41.4463	0.0241	9.6036	0.1041
20	4.6610	0.2145	45.7620	0.0219	9.8181	0.1019
21	5.0338	0.1987	50.4229	0.0198	10.0168	0.0998
22	5.4365	0.1839	55.4568	0.0180	10.2007	0.0980
23	5.8715	0.1703	60.8933	0.0164	10.3711	0.0964
24	6.3412	0.1577	66.7648	0.0150	10.5288	0.0950
25	6.8485	0.1460	73.1059	0.0137	10.6748	0.0937

	$(1+i)^n$	$\dfrac{1}{(1+i)^n}$	$\dfrac{(1+i)^n-1}{i}$	$\dfrac{i}{(1+i)^n-1}$	$\dfrac{(1+i)^n-1}{i(1+i)^n}$	$\dfrac{i(1+i)^n}{(1+i)^n-1}$
n			**i = 9%**			
1	1.0900	0.9174	1.0000	1.0000	0.9174	1.0900
2	1.1881	0.8417	2.0900	0.4785	1.7591	0.5685
3	1.2950	0.7722	3.2781	0.3051	2.5313	0.3951
4	1.4116	0.7084	4.5731	0.2187	3.2397	0.3087
5	1.5386	0.6499	5.9847	0.1671	3.8897	0.2571
6	1.6771	0.5963	7.5233	0.1329	4.4859	0.2229
7	1.8280	0.5470	9.2004	0.1087	5.0330	0.1987
8	1.9926	0.5019	11.0285	0.0907	5.5348	0.1807
9	2.1719	0.4604	13.0210	0.0768	5.9952	0.1668
10	2.3674	0.4224	15.1929	0.0658	6.4177	0.1558
11	2.5804	0.3875	17.5603	0.0569	6.8052	0.1469
12	2.8127	0.3555	20.1407	0.0497	7.1607	0.1397
13	3.0658	0.3262	22.9534	0.0436	7.4869	0.1336
14	3.3417	0.2992	26.0192	0.0384	7.7862	0.1284
15	3.6425	0.2745	29.3609	0.0341	8.0607	0.1241
16	3.9703	0.2519	33.0034	0.0303	8.3126	0.1203
17	4.3276	0.2311	36.9737	0.0270	8.5436	0.1170
18	4.7171	0.2120	41.3013	0.0242	8.7556	0.1142
19	5.1417	0.1945	46.0185	0.0217	8.9501	0.1117
20	5.6044	0.1784	51.1601	0.0195	9.1285	0.1095
21	6.1088	0.1637	56.7645	0.0176	9.2922	0.1076
22	6.6586	0.1502	62.8733	0.0159	9.4424	0.1059
23	7.2579	0.1378	69.5319	0.0144	9.5802	0.1044
24	7.9111	0.1264	76.7898	0.0130	9.7066	0.1030
25	8.6231	0.1160	84.7009	0.0118	9.8226	0.1018
n			**i = 10%**			
1	1.1000	0.9091	1.0000	1.0000	0.9091	1.1000
2	1.2100	0.8264	2.1000	0.4762	1.7355	0.5762
3	1.3310	0.7513	3.3100	0.3021	2.4869	0.4021
4	1.4641	0.6830	4.6410	0.2155	3.1699	0.3155
5	1.6105	0.6209	6.1051	0.1638	3.7908	0.2638
6	1.7716	0.5645	7.7156	0.1296	4.3553	0.2296
7	1.9487	0.5132	9.4872	0.1054	4.8684	0.2054
8	2.1436	0.4665	11.4359	0.0874	5.3349	0.1874
9	2.3579	0.4241	13.5795	0.0736	5.7590	0.1736
10	2.5937	0.3855	15.9374	0.0627	6.1446	0.1627
11	2.8531	0.3505	18.5312	0.0540	6.4951	0.1540
12	3.1384	0.3186	21.3843	0.0468	6.8137	0.1468
13	3.4523	0.2897	24.5227	0.0408	7.1034	0.1408
14	3.7975	0.2633	27.9750	0.0357	7.3667	0.1357
15	4.1772	0.2394	31.7725	0.0315	7.6061	0.1315
16	4.5950	0.2176	35.9497	0.0278	7.8237	0.1278
17	5.0545	0.1978	40.5447	0.0247	8.0216	0.1247
18	5.5599	0.1799	45.5992	0.0219	8.2014	0.1219
19	6.1159	0.1635	51.1591	0.0195	8.3649	0.1195
20	6.7275	0.1486	57.2750	0.0175	8.5136	0.1175
21	7.4002	0.1351	64.0025	0.0156	8.6487	0.1156
22	8.1403	0.1228	71.4027	0.0140	8.7715	0.1140
23	8.9543	0.1117	79.5430	0.0126	8.8832	0.1126
24	9.8497	0.1015	88.4973	0.0113	8.9847	0.1113
25	10.8347	0.0923	98.3471	0.0102	9.0770	0.1102

	$(1+i)^n$	$\dfrac{1}{(1+i)^n}$	$\dfrac{(1+i)^n-1}{i}$	$\dfrac{i}{(1+i)^n-1}$	$\dfrac{(1+i)^n-1}{i(1+i)^n}$	$\dfrac{i(1+i)^n}{(1+i)^n-1}$
n			**i = 11%**			
1	1.1100	0.9009	1.0000	1.0000	0.9009	1.1100
2	1.2321	0.8116	2.1100	0.4739	1.7125	0.5839
3	1.3676	0.7312	3.3421	0.2992	2.4437	0.4092
4	1.5181	0.6587	4.7097	0.2123	3.1024	0.3223
5	1.6851	0.5935	6.2278	0.1606	3.6959	0.2706
6	1.8704	0.5346	7.9129	0.1264	4.2305	0.2364
7	2.0762	0.4817	9.7833	0.1022	4.7122	0.2122
8	2.3045	0.4339	11.8594	0.0843	5.1461	0.1943
9	2.5580	0.3909	14.1640	0.0706	5.5370	0.1806
10	2.8394	0.3522	16.7220	0.0598	5.8892	0.1698
11	3.1518	0.3173	19.5614	0.0511	6.2065	0.1611
12	3.4985	0.2858	22.7132	0.0440	6.4924	0.1540
13	3.8833	0.2575	26.2116	0.0382	6.7499	0.1482
14	4.3104	0.2320	30.0949	0.0332	6.9819	0.1432
15	4.7846	0.2090	34.4054	0.0291	7.1909	0.1391
16	5.3109	0.1883	39.1899	0.0255	7.3792	0.1355
17	5.8951	0.1696	44.5008	0.0225	7.5488	0.1325
18	6.5436	0.1528	50.3959	0.0198	7.7016	0.1298
19	7.2633	0.1377	56.9395	0.0176	7.8393	0.1276
20	8.0623	0.1240	64.2028	0.0156	7.9633	0.1256
21	8.9492	0.1117	72.2651	0.0138	8.0751	0.1238
22	9.9336	0.1007	81.2143	0.0123	8.1757	0.1223
23	11.0263	0.0907	91.1479	0.0110	8.2664	0.1210
24	12.2392	0.0817	102.1742	0.0098	8.3481	0.1198
25	13.5855	0.0736	114.4133	0.0087	8.4217	0.1187
n			**i = 12%**			
1	1.1200	0.8929	1.0000	1.0000	0.8929	1.1200
2	1.2544	0.7972	2.1200	0.4717	1.6901	0.5917
3	1.4049	0.7118	3.3744	0.2963	2.4018	0.4163
4	1.5735	0.6355	4.7793	0.2092	3.0373	0.3292
5	1.7623	0.5674	6.3528	0.1574	3.6048	0.2774
6	1.9738	0.5066	8.1152	0.1232	4.1114	0.2432
7	2.2107	0.4523	10.0890	0.0991	4.5638	0.2191
8	2.4760	0.4039	12.2997	0.0813	4.9676	0.2013
9	2.7731	0.3606	14.7757	0.0677	5.3282	0.1877
10	3.1058	0.3220	17.5487	0.0570	5.6502	0.1770
11	3.4785	0.2875	20.6546	0.0484	5.9377	0.1684
12	3.8960	0.2567	24.1331	0.0414	6.1944	0.1614
13	4.3635	0.2292	28.0291	0.0357	6.4235	0.1557
14	4.8871	0.2046	32.3926	0.0309	6.6282	0.1509
15	5.4736	0.1827	37.2797	0.0268	6.8109	0.1468
16	6.1304	0.1631	42.7533	0.0234	6.9740	0.1434
17	6.8660	0.1456	48.8837	0.0205	7.1196	0.1405
18	7.6900	0.1300	55.7497	0.0179	7.2497	0.1379
19	8.6128	0.1161	63.4397	0.0158	7.3658	0.1358
20	9.6463	0.1037	72.0524	0.0139	7.4694	0.1339
21	10.8038	0.0926	81.6987	0.0122	7.5620	0.1322
22	12.1003	0.0826	92.5026	0.0108	7.6446	0.1308
23	13.5523	0.0738	104.6029	0.0096	7.7184	0.1296
24	15.1786	0.0659	118.1552	0.0085	7.7843	0.1285
25	17.0001	0.0588	133.3339	0.0075	7.8431	0.1275

	$(1+i)^n$	$\dfrac{1}{(1+i)^n}$	$\dfrac{(1+i)^n-1}{i}$	$\dfrac{i}{(1+i)^n-1}$	$\dfrac{(1+i)^n-1}{i(1+i)^n}$	$\dfrac{i(1+i)^n}{(1+i)^n-1}$
n			$i=13\%$			
1	1.1300	0.8850	1.0000	1.0000	0.8850	1.1300
2	1.2769	0.7831	2.1300	0.4695	1.6681	0.5995
3	1.4429	0.6931	3.4069	0.2935	2.3612	0.4235
4	1.6305	0.6133	4.8498	0.2062	2.9745	0.3362
5	1.8424	0.5428	6.4803	0.1543	3.5172	0.2843
6	2.0820	0.4803	8.3227	0.1202	3.9975	0.2502
7	2.3526	0.4251	10.4047	0.0961	4.4226	0.2261
8	2.6584	0.3762	12.7573	0.0784	4.7988	0.2084
9	3.0040	0.3329	15.4157	0.0649	5.1317	0.1949
10	3.3946	0.2946	18.4197	0.0543	5.4262	0.1843
11	3.8359	0.2607	21.8143	0.0458	5.6869	0.1758
12	4.3345	0.2307	25.6502	0.0390	5.9176	0.1690
13	4.8980	0.2042	29.9847	0.0334	6.1218	0.1634
14	5.5348	0.1807	34.8827	0.0287	6.3025	0.1587
15	6.2543	0.1599	40.4175	0.0247	6.4624	0.1547
16	7.0673	0.1415	46.6717	0.0214	6.6039	0.1514
17	7.9861	0.1252	53.7391	0.0186	6.7291	0.1486
18	9.0243	0.1108	61.7251	0.0162	6.8399	0.1462
19	10.1974	0.0981	70.7494	0.0141	6.9380	0.1441
20	11.5231	0.0868	80.9468	0.0124	7.0248	0.1424
21	13.0211	0.0768	92.4699	0.0108	7.1016	0.1408
22	14.7138	0.0680	105.4910	0.0095	7.1695	0.1395
23	16.6266	0.0601	120.2048	0.0083	7.2297	0.1383
24	18.7881	0.0532	136.8315	0.0073	7.2829	0.1373
25	21.2305	0.0471	155.6196	0.0064	7.3300	0.1364
n			$i=14\%$			
1	1.1400	0.8772	1.0000	1.0000	0.8772	1.1400
2	1.2996	0.7695	2.1400	0.4673	1.6467	0.6073
3	1.4815	0.6750	3.4396	0.2907	2.3216	0.4307
4	1.6890	0.5921	4.9211	0.2032	2.9137	0.3432
5	1.9254	0.5194	6.6101	0.1513	3.4331	0.2913
6	2.1950	0.4556	8.5355	0.1172	3.8887	0.2572
7	2.5023	0.3996	10.7305	0.0932	4.2883	0.2332
8	2.8526	0.3506	13.2328	0.0756	4.6389	0.2156
9	3.2519	0.3075	16.0853	0.0622	4.9464	0.2022
10	3.7072	0.2697	19.3373	0.0517	5.2161	0.1917
11	4.2262	0.2366	23.0445	0.0434	5.4527	0.1834
12	4.8179	0.2076	27.2707	0.0367	5.6603	0.1767
13	5.4924	0.1821	32.0887	0.0312	5.8424	0.1712
14	6.2613	0.1597	37.5811	0.0266	6.0021	0.1666
15	7.1379	0.1401	43.8424	0.0228	6.1422	0.1628
16	8.1372	0.1229	50.9804	0.0196	6.2651	0.1596
17	9.2765	0.1078	59.1176	0.0169	6.3729	0.1569
18	10.5752	0.0946	68.3941	0.0146	6.4674	0.1546
19	12.0557	0.0829	78.9692	0.0127	6.5504	0.1527
20	13.7435	0.0728	91.0249	0.0110	6.6231	0.1510
21	15.6676	0.0638	104.7684	0.0095	6.6870	0.1495
22	17.8610	0.0560	120.4360	0.0083	6.7429	0.1483
23	20.3616	0.0491	138.2970	0.0072	6.7921	0.1472
24	23.2122	0.0431	158.6586	0.0063	6.8351	0.1463
25	26.4619	0.0378	181.8708	0.0055	6.8729	0.1455

	$(1+i)^n$	$\dfrac{1}{(1+i)^n}$	$\dfrac{(1+i)^n-1}{i}$	$\dfrac{i}{(1+i)^n-1}$	$\dfrac{(1+i)^n-1}{i(1+i)^n}$	$\dfrac{i(1+i)^n}{(1+i)^n-1}$
n			$i = 15\%$			
1	1.1500	0.8696	1.0000	1.0000	0.8696	1.1500
2	1.3225	0.7561	2.1500	0.4651	1.6257	0.6151
3	1.5209	0.6575	3.4725	0.2880	2.2832	0.4380
4	1.7490	0.5718	4.9934	0.2003	2.8550	0.3503
5	2.0114	0.4972	6.7424	0.1483	3.3522	0.2983
6	2.3131	0.4323	8.7537	0.1142	3.7845	0.2642
7	2.6600	0.3759	11.0668	0.0904	4.1604	0.2404
8	3.0590	0.3269	13.7268	0.0729	4.4873	0.2229
9	3.5179	0.2843	16.7858	0.0596	4.7716	0.2096
10	4.0456	0.2472	20.3037	0.0493	5.0188	0.1993
11	4.6524	0.2149	24.3493	0.0411	5.2337	0.1911
12	5.3503	0.1869	29.0017	0.0345	5.4206	0.1845
13	6.1528	0.1625	34.3519	0.0291	5.5831	0.1791
14	7.0757	0.1413	40.5047	0.0247	5.7245	0.1747
15	8.1371	0.1229	47.5804	0.0210	5.8474	0.1710
16	9.3576	0.1069	55.7175	0.0179	5.9542	0.1679
17	10.7613	0.0929	65.0751	0.0154	6.0472	0.1654
18	12.3755	0.0808	75.8364	0.0132	6.1280	0.1632
19	14.2318	0.0703	88.2118	0.0113	6.1982	0.1613
20	16.3665	0.0611	102.4436	0.0098	6.2593	0.1598
21	18.8215	0.0531	118.8101	0.0084	6.3125	0.1584
22	21.6447	0.0462	137.6316	0.0073	6.3587	0.1573
23	24.8915	0.0402	159.2764	0.0063	6.3988	0.1563
24	28.6252	0.0349	184.1678	0.0054	6.4338	0.1554
25	32.9190	0.0304	212.7930	0.0047	6.4641	0.1547
n			$i = 16\%$			
1	1.1600	0.8621	1.0000	1.0000	0.8621	1.1600
2	1.3456	0.7432	2.1600	0.4630	1.6052	0.6230
3	1.5609	0.6407	3.5056	0.2853	2.2459	0.4453
4	1.8106	0.5523	5.0665	0.1974	2.7982	0.3574
5	2.1003	0.4761	6.8771	0.1454	3.2743	0.3054
6	2.4364	0.4104	8.9775	0.1114	3.6847	0.2714
7	2.8262	0.3538	11.4139	0.0876	4.0386	0.2476
8	3.2784	0.3050	14.2401	0.0702	4.3436	0.2302
9	3.8030	0.2630	17.5185	0.0571	4.6065	0.2171
10	4.4114	0.2267	21.3215	0.0469	4.8332	0.2069
11	5.1173	0.1954	25.7329	0.0389	5.0286	0.1989
12	5.9360	0.1685	30.8502	0.0324	5.1971	0.1924
13	6.8858	0.1452	36.7862	0.0272	5.3423	0.1872
14	7.9875	0.1252	43.6720	0.0229	5.4675	0.1829
15	9.2655	0.1079	51.6595	0.0194	5.5755	0.1794
16	10.7480	0.0930	60.9250	0.0164	5.6685	0.1764
17	12.4677	0.0802	71.6730	0.0140	5.7487	0.1740
18	14.4625	0.0691	84.1407	0.0119	5.8178	0.1719
19	16.7765	0.0596	98.6032	0.0101	5.8775	0.1701
20	19.4608	0.0514	115.3797	0.0087	5.9288	0.1687
21	22.5745	0.0443	134.8405	0.0074	5.9731	0.1674
22	26.1864	0.0382	157.4150	0.0064	6.0113	0.1664
23	30.3762	0.0329	183.6014	0.0054	6.0442	0.1654
24	35.2364	0.0284	213.9776	0.0047	6.0726	0.1647
25	40.8742	0.0245	249.2140	0.0040	6.0971	0.1640

	$(1+i)^n$	$\dfrac{1}{(1+i)^n}$	$\dfrac{(1+i)^n-1}{i}$	$\dfrac{i}{(1+i)^n-1}$	$\dfrac{(1+i)^n-1}{i(1+i)^n}$	$\dfrac{i(1+i)^n}{(1+i)^n-1}$
n			**i = 17%**			
1	1.1700	0.8547	1.0000	1.0000	0.8547	1.1700
2	1.3689	0.7305	2.1700	0.4608	1.5852	0.6308
3	1.6016	0.6244	3.5389	0.2826	2.2096	0.4526
4	1.8739	0.5337	5.1405	0.1945	2.7432	0.3645
5	2.1924	0.4561	7.0144	0.1426	3.1993	0.3126
6	2.5652	0.3898	9.2068	0.1086	3.5892	0.2786
7	3.0012	0.3332	11.7720	0.0849	3.9224	0.2549
8	3.5115	0.2848	14.7733	0.0677	4.2072	0.2377
9	4.1084	0.2434	18.2847	0.0547	4.4506	0.2247
10	4.8068	0.2080	22.3931	0.0447	4.6586	0.2147
11	5.6240	0.1778	27.1999	0.0368	4.8364	0.2068
12	6.5801	0.1520	32.8239	0.0305	4.9884	0.2005
13	7.6987	0.1299	39.4040	0.0254	5.1183	0.1954
14	9.0075	0.1110	47.1027	0.0212	5.2293	0.1912
15	10.5387	0.0949	56.1101	0.0178	5.3242	0.1878
16	12.3303	0.0811	66.6488	0.0150	5.4053	0.1850
17	14.4265	0.0693	78.9792	0.0127	5.4746	0.1827
18	16.8790	0.0592	93.4056	0.0107	5.5339	0.1807
19	19.7484	0.0506	110.2846	0.0091	5.5845	0.1791
20	23.1056	0.0433	130.0329	0.0077	5.6278	0.1777
21	27.0336	0.0370	153.1385	0.0065	5.6648	0.1765
22	31.6293	0.0316	180.1721	0.0056	5.6964	0.1756
23	37.0062	0.0270	211.8013	0.0047	5.7234	0.1747
24	43.2973	0.0231	248.8076	0.0040	5.7465	0.1740
25	50.6578	0.0197	292.1049	0.0034	5.7662	0.1734
n			**i = 18%**			
1	1.1800	0.8475	1.0000	1.0000	0.8475	1.1800
2	1.3924	0.7182	2.1800	0.4587	1.5656	0.6387
3	1.6430	0.6086	3.5724	0.2799	2.1743	0.4599
4	1.9388	0.5158	5.2154	0.1917	2.6901	0.3717
5	2.2878	0.4371	7.1542	0.1398	3.1272	0.3198
6	2.6996	0.3704	9.4420	0.1059	3.4976	0.2859
7	3.1855	0.3139	12.1415	0.0824	3.8115	0.2624
8	3.7589	0.2660	15.3270	0.0652	4.0776	0.2452
9	4.4355	0.2255	19.0859	0.0524	4.3030	0.2324
10	5.2338	0.1911	23.5213	0.0425	4.4941	0.2225
11	6.1759	0.1619	28.7551	0.0348	4.6560	0.2148
12	7.2876	0.1372	34.9311	0.0286	4.7932	0.2086
13	8.5994	0.1163	42.2187	0.0237	4.9095	0.2037
14	10.1472	0.0985	50.8180	0.0197	5.0081	0.1997
15	11.9737	0.0835	60.9653	0.0164	5.0916	0.1964
16	14.1290	0.0708	72.9390	0.0137	5.1624	0.1937
17	16.6722	0.0600	87.0680	0.0115	5.2223	0.1915
18	19.6733	0.0508	103.7403	0.0096	5.2732	0.1896
19	23.2144	0.0431	123.4135	0.0081	5.3162	0.1881
20	27.3930	0.0365	146.6280	0.0068	5.3527	0.1868
21	32.3238	0.0309	174.0210	0.0057	5.3837	0.1857
22	38.1421	0.0262	206.3448	0.0048	5.4099	0.1848
23	45.0076	0.0222	244.4868	0.0041	5.4321	0.1841
24	53.1090	0.0188	289.4945	0.0035	5.4509	0.1835
25	62.6686	0.0160	342.6035	0.0029	5.4669	0.1829

	$(1+i)^n$	$\dfrac{1}{(1+i)^n}$	$\dfrac{(1+i)^n-1}{i}$	$\dfrac{i}{(1+i)^n-1}$	$\dfrac{(1+i)^n-1}{i(1+i)^n}$	$\dfrac{i(1+i)^n}{(1+i)^n-1}$
n			$i = 19\%$			
1	1.1900	0.8403	1.0000	1.0000	0.8403	1.1900
2	1.4161	0.7062	2.1900	0.4566	1.5465	0.6466
3	1.6852	0.5934	3.6061	0.2773	2.1399	0.4673
4	2.0053	0.4987	5.2913	0.1890	2.6386	0.3790
5	2.3864	0.4190	7.2966	0.1371	3.0576	0.3271
6	2.8398	0.3521	9.6830	0.1033	3.4098	0.2933
7	3.3793	0.2959	12.5227	0.0799	3.7057	0.2699
8	4.0214	0.2487	15.9020	0.0629	3.9544	0.2529
9	4.7854	0.2090	19.9234	0.0502	4.1633	0.2402
10	5.6947	0.1756	24.7089	0.0405	4.3389	0.2305
11	6.7767	0.1476	30.4035	0.0329	4.4865	0.2229
12	8.0642	0.1240	37.1802	0.0269	4.6105	0.2169
13	9.5964	0.1042	45.2445	0.0221	4.7147	0.2121
14	11.4198	0.0876	54.8409	0.0182	4.8023	0.2082
15	13.5895	0.0736	66.2607	0.0151	4.8759	0.2051
16	16.1715	0.0618	79.8502	0.0125	4.9377	0.2025
17	19.2441	0.0520	96.0218	0.0104	4.9897	0.2004
18	22.9005	0.0437	115.2659	0.0087	5.0333	0.1987
19	27.2516	0.0367	138.1664	0.0072	5.0700	0.1972
20	32.4294	0.0308	165.4180	0.0060	5.1009	0.1960
21	38.5910	0.0259	197.8474	0.0051	5.1268	0.1951
22	45.9233	0.0218	236.4385	0.0042	5.1486	0.1942
23	54.6487	0.0183	282.3618	0.0035	5.1668	0.1935
24	65.0320	0.0154	337.0105	0.0030	5.1822	0.1930
25	77.3881	0.0129	402.0425	0.0025	5.1951	0.1925
n			$i = 20\%$			
1	1.2000	0.8333	1.0000	1.0000	0.8333	1.2000
2	1.4400	0.6944	2.2000	0.4545	1.5278	0.6545
3	1.7280	0.5787	3.6400	0.2747	2.1065	0.4747
4	2.0736	0.4823	5.3680	0.1863	2.5887	0.3863
5	2.4883	0.4019	7.4416	0.1344	2.9906	0.3344
6	2.9860	0.3349	9.9299	0.1007	3.3255	0.3007
7	3.5832	0.2791	12.9159	0.0774	3.6046	0.2774
8	4.2998	0.2326	16.4991	0.0606	3.8372	0.2606
9	5.1598	0.1938	20.7989	0.0481	4.0310	0.2481
10	6.1917	0.1615	25.9587	0.0385	4.1925	0.2385
11	7.4301	0.1346	32.1504	0.0311	4.3271	0.2311
12	8.9161	0.1122	39.5805	0.0253	4.4392	0.2253
13	10.6993	0.0935	48.4966	0.0206	4.5327	0.2206
14	12.8392	0.0779	59.1959	0.0169	4.6106	0.2169
15	15.4070	0.0649	72.0351	0.0139	4.6755	0.2139
16	18.4884	0.0541	87.4421	0.0114	4.7296	0.2114
17	22.1861	0.0451	105.9306	0.0094	4.7746	0.2094
18	26.6233	0.0376	128.1167	0.0078	4.8122	0.2078
19	31.9480	0.0313	154.7400	0.0065	4.8435	0.2065
20	38.3376	0.0261	186.6880	0.0054	4.8696	0.2054
21	46.0051	0.0217	225.0256	0.0044	4.8913	0.2044
22	55.2061	0.0181	271.0307	0.0037	4.9094	0.2037
23	66.2474	0.0151	326.2369	0.0031	4.9245	0.2031
24	79.4968	0.0126	392.4842	0.0025	4.9371	0.2025
25	95.3962	0.0105	471.9811	0.0021	4.9476	0.2021

10 Endnotes

1   (Price/unit × Units sold in 1st year) / 12 = ($100 × 105,000) / 12 = $875,000

2   (Cost of raw material/unit × Units sold) / 12 = ($20 × 105,000) / 12 = $175,000

3   0.5 month of finished goods = 0.5 × $380,000 = $190,000

4   (Variable cost/unit × Units sold) / 12 = ($43.33 × 105,000) /12 = $380,000

5   Similar to Endnote 2.